SIMON FRASER UNIVERSITY

MATH 155 Midterm 1

7 February 2007, 08:30–09:20

Last Name _____

Given Name(s) _____

Student # _____

Signature _____

INSTRUCTIONS

1. **Do not open this booklet until told to do so**.
2. Write your last name, given name(s), and student number in the box above. Sign on the last line of the box.
3. This exam has 6 questions on 5 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden**.
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	7	
2	8	
3	5	
4	5	
5	6	
6	8	
Total	39	

1. Clearly indicate if the following statements are true (T) or false (F).

Assume that all functions are continuous in the intervals of integration.

A statement containing general constants and/or functions (f, g, a, b, c, n) is true if and only if it holds for *all admissible choices* of these constants and/or functions.

[1] (a) _____ $\sum_{k=1}^{n}(k+1) = \dfrac{n(n+3)}{2}$

[1] (b) _____ $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right) \cdot \left(\int_a^b g(x)\,dx\right)$

[1] (c) _____ If $\int_a^b f(x)\,dx \geq 0$, then $f(x) \geq 0$ for all x in $[a, b]$.

[1] (d) _____ If $f(x) \geq c$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq c$.

[1] (e) _____ $\int_0^1 x \sin x\,dx \leq \dfrac{1}{2}$

[1] (f) _____ $\int_a^b [f(x) - g(x)]\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$

[1] (g) _____ $\int_a^b f(x)\,dx = \left(\int_a^c f(x)\,dx\right) - \left(\int_b^c f(x)\,dx\right)$

[5] **2.** (a) Let $L(x)$ denote the length of a certain organism at age x ($x \geq 0$). At the moment of birth (age $x = 0$) the length of the organism was $L(0) = 5$. By studying literature you learned that $L(x)$ is governed by

$$\frac{dL}{dx} = e^{-x/10}.$$

Find $L(x)$ for each $x \geq 0$.

[3] (b) There is a certain length L_∞ which the organism will approach at maturity (at very large age). Determine L_∞.

[5] **3.** Determine the average value of $f(x) = \dfrac{3}{\sqrt{1-x^2}}$ in the interval $[0, 1/2]$.

[5] **4.** Express the area between the curves $y = 2x^2 + 4$ and $y = 7 - x^2$ as a definite integral. **DO NOT EVALUATE THE INTEGRAL.**

[3] **5.** (a) Describe the right-circular cone with base radius r and height h as a solid of revolution.

[3] (b) Use the formula for the volume of a solid of revolution to express the volume of the right-circular cone with base radius r and height h as a definite integral. **DO NOT EVALUATE THE INTEGRAL.**

[8] **6.** Evaluate $\displaystyle\int_2^3 \frac{2x^3}{\sqrt{x^2-1}}\,dx$. Hint: Write $2x^3 = 2x \cdot x^2$.