GREEN AND BLUE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 1

6 February 2008, 08:30–09:20

Last Name ____________

Given Name(s) ____________

Student # ____________

Signature ____________

INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Write your last name, given name(s), and student number in the box above. Sign on the last line of the box.
3. This exam has 6 questions on 5 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.**
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	7	
2	8	
3	5	
4	4	
5	8	
6	8	
Total	40	

1. Indicate whether the following statements are true (T) or false (F). Justifications are **not required.**

Assume that $f(x)$ is continuous in the intervals of integration.

A statement containing general constants a, b, c and function f is true if and only if it holds for *all admissible choices* that you can make for a, b, c, f.

[1] (a) F $\sum_{k=1}^{10} f(k) = \sum_{k=1}^{5} f(k) + \sum_{k=5}^{10} f(k)$ $f(5)$ included twice

[1] (b) F If $\int_a^b f(x)\,dx \geq 0$, then $f(x) \geq 0$ for all x in $[a, b]$. 

[1] (c) T $\int_a^b f(x)\,dx = \int_c^b f(x)\,dx - \int_c^a f(x)\,dx$ Sec 6.1.3, Property 5

[1] (d) F $\int_{-1}^{0} (e^x)^2\,dx \geq \int_{-1}^{0} e^x\,dx$ $x<0 \Rightarrow e^x < 1, \Rightarrow (e^x)^2 < e^x$
$e^x > 0$

[1] (e) T $\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = 0$ Sec. 6.1.3, Property 2

[1] (f) T $\int_{-1}^{0} \tan x\,dx = -\int_0^1 \tan x\,dx$ graph of $\tan x$ is symmetric about the origin

[1] (g) F If $f(x) \leq c$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \leq c(a+b)$.
needs $c(b-a)$

[5] 2. (a) Let $N(t)$ denote the size (number of members) of a population at time t ($t \geq 0$). It is given that $N(0) = 110$ and $N(t)$ is governed by

$$\frac{dN}{dt} = 60e^{-t}.$$

Find the size of the population at time $t = 1$.

Sec. 5.8

$N(t) = -60e^{-t} + C$

$110 = -60 + C$

$C = 170$

$N(t) = -60e^{-t} + 170$

or

Sec. 6.3.2

$N(t) = N(0) + \int_0^t 60e^{-u}\,du$

$= 110 + \left[-60e^{-u}\right]_0^t$

$= -60e^{-t} + 170$

$N(1) = -60e^{-1} + 170 \approx 147.93$

[3] (b) At very large time t the population size will stabilize around a certain value N_∞. Determine N_∞.

$$N_\infty = \lim_{t\to\infty} N(t) = \lim_{t\to\infty}\left(-60e^{-t} + 170\right) =$$

$$= 0 + 170 = 170$$

[5] 3. Determine the average value of $f(x) = \dfrac{2}{7(1+x^2)}$ in the interval $[-1, 1]$.

$$f_{avg} = \frac{1}{1-(-1)} \int_{-1}^{1} \frac{2\,dx}{7(1+x^2)} = \frac{1}{2}\left[\frac{2}{7}\tan^{-1}x\right]_{-1}^{1} =$$

$$= \frac{1}{2}\left(\frac{2}{7}\cdot\frac{\pi}{4} - \frac{2}{7}\cdot\left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{14} \approx 0.22$$

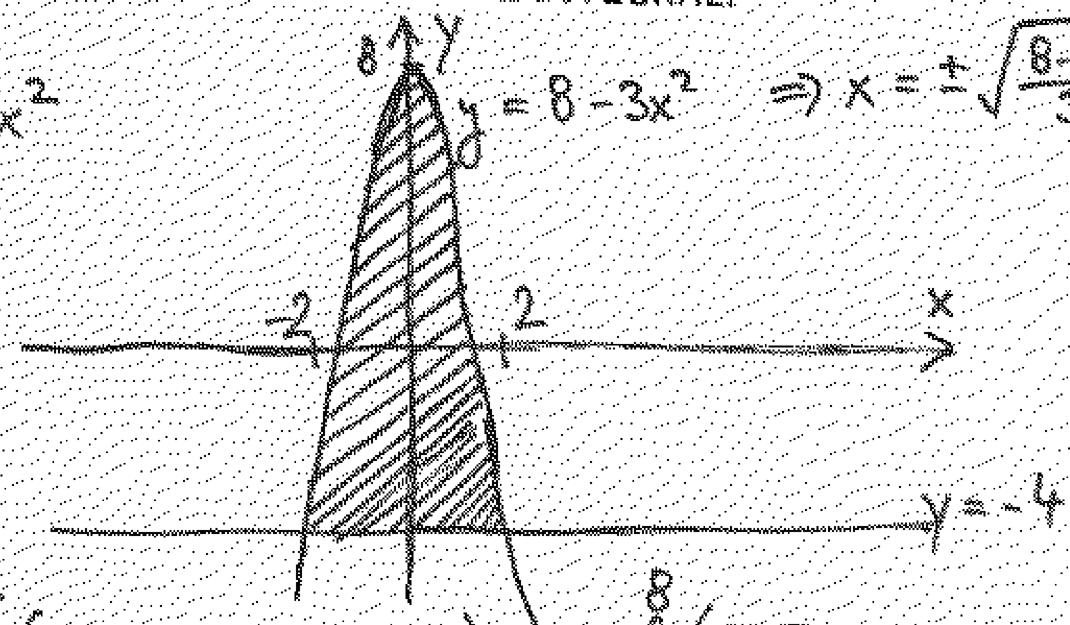
[4] 4. Express the area between the curves $y = -4$ and $y = 8 - 3x^2$ as a definite integral. **DO NOT EVALUATE THE INTEGRAL.**

$-4 = 8 - 3x^2$

$3x^2 = 12$

$x = \pm 2$

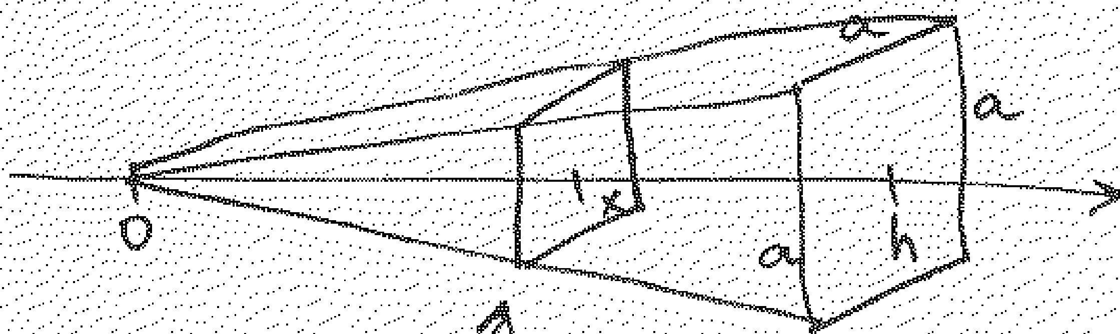
$y = 8 - 3x^2 \Rightarrow x = \pm\sqrt{\dfrac{8-y}{3}}$



$$\text{Area} = \int_{-2}^{2} \left((8-3x^2) - (-4)\right) dx = \int_{-4}^{8} \left(\sqrt{\frac{8-y}{3}} - \left(-\sqrt{\frac{8-y}{3}}\right)\right) dy$$

EITHER FORMULA IS SUFFICIENT.

[8] 5. Use an integral to find the volume of a pyramid with square base of side length a and height h. Show the details of your computation. Evaluate the integral.



The cross-section at point x is a square of side length $\frac{a}{h}x$.

$$\text{Volume} = \int_0^h \left(\frac{a}{h}x\right)^2 dx = \left[\frac{a^2}{h^2}\frac{x^3}{3}\right]_0^h =$$

$$= \frac{a^2}{h^2}\left(\frac{h^3}{3} - 0\right) = \frac{a^2 h}{3}$$

Please see the posted solution to HW 6.3.34 for details.

[8] 6. Evaluate $\int_0^3 (x-1)\sqrt{1+x}\,dx$.

$$u = 1 + x \Rightarrow x - 1 = u - 2$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int_0^3 (x-1)\sqrt{1+x}\,dx = \int_1^4 (u-2)\sqrt{u}\,du =$$

$$= \int_1^4 \left(u^{3/2} - 2u^{1/2}\right) du = \left[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}\right]_1^4 =$$

$$= \left(\frac{2}{5}\cdot 4^{\frac{5}{2}} - \frac{4}{3}\cdot 4^{\frac{3}{2}}\right) - \left(\frac{2}{5}\cdot 1^{\frac{5}{2}} - \frac{4}{3}\cdot 1^{\frac{3}{2}}\right) =$$

$$= \frac{2}{5}\cdot 32 - \frac{4}{3}\cdot 8 - \frac{2}{5} + \frac{4}{3} = \frac{62}{5} - \frac{28}{3} = \frac{46}{15}$$

$$\approx 3.07$$