Simon Fraser University
MATH 155 – FINAL EXAM
Instructor: N.Kouzniak

August 5, 2005

8.30-11.30

Last Name______________________________

Given Name(s)_________________________

Student ID______________________________

Signature___________________________

INSTRUCTIONS

1. Do not open this booklet until instructed to do so. The booklet contains 14 pages including the cover page.

2. Print your name and student ID in the space provided above.

3. For each question you must **show all your work** unless stated otherwise.

4. No book, paper, or device other than the usual writing instruments, this booklet, and scientific calculators are allowed. **In particular, no graphing/programmable calculators are allowed.**

5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.

6. You may use the back of the previous page for a rough work or if you run out of space.

7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Question	Maximum	Mark
1	4	
2	18	
3	8	
4	4	
5	6	
6	4	
7	10	
8	8	
9	8	
10	8	
11	7	
12	5	
13	4	
14	6	
Total	100	

1. [4 marks total] Based on geometrical interpretation and properties of the definite integrals, prove without evaluating the integrals:

a) [2 marks] $\int_{-2}^{2}(1-|x|)dx = 0.$

b) [2 marks] $0.5 \le \int_{0}^{1}\sqrt{1-x^2dx} \le 1.$

2. [18 marks total] Evaluate the following integrals

a) [5 marks] $\int e^{3x}\cos 5x dx$

b) [3 marks] $\int \frac{dx}{x^2-2x+2}$

c) [5 marks] $\int_1^e \frac{dx}{x\sqrt{\ln x}}$

d) [5 marks] $\int_1^\infty \frac{\ln x dx}{x^2}$

3. [8 marks total] Set up (but **do not evaluate**!) the integrals required to find the following quantities:

a) [3 marks] Volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = \frac{1}{x^2}$, $x = 0$, $y = 1$, $y = 4$ in the first quadrant.

b) [3 marks] Area of the region enclosed between the curves $y = |x|$, $y = x^2 - 2$.

c) [2 marks] Average value of the function $y = \tan x$, $x \in [-\pi/4, \pi/4]$. Where is it achieved in the interval?

4. [4 marks total] Use the following numerical methods of integration with $n = 4$ to approximate $\int_0^1 x^2 dx$:

a) [2 marks] The midpoint rule

b) [2 marks] The trapezoidal rule

5. [6 marks total] a) [3 marks] Find Taylor polynomial of degree 5 about $a=0$ for $f(x)=\cos x$

b) [3 marks] Use your result in part a) to find $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$

6. [4 marks] Find function $f(x)$ such that

$$6+\int_a^x \frac{f(t)dt}{t^2}=2\sqrt{x}.$$

7. [10 marks total] Assume that the size of a population evolves according to the logistic equation $\frac{dN}{dt} = 1.5N(1 - \frac{N}{100})$.

a) [4 marks] Find all equilibria and discuss their stability using the eigenvalues.

b) [6 marks] Solve differential equation if $N(0) = 120$.

8. [8 marks total] Find all meaningful equilibria of the differential equations and discuss their stability using graphical approach.

a) [4 marks] $\dfrac{dy}{dx} = y(1-y)(y-2).$

b) [4 marks] $\dfrac{dp}{dt} = 0.5p(1-p) - 1.5p$ where $p(t)$ is the fraction of occupied patched in a metapopulation described by Levins model.

9. [8 marks total] A linear system which contains a parameter a is given by

$$\begin{cases} -2x - 2z + y = 0 \\ x - 2y - 2z = 1 \\ -2x + az - 2y = 0 \end{cases}$$

a) [4 marks] Reduce the system to the upper triangular form

b) [2 mark] Determine for what value of a the system does not have a solution.

c) [2 marks] Determine for what values of a the system has exactly one solution and find it if a= -5.

10. [8 marks total] Given the matrix

$$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -2 & 0 \\ -1 & 1 & 2 \end{bmatrix},$$

a) [3 marks] Prove that it is non-singular.

b) [5 marks] Find the inverse A^{-1}.

11. [7 marks total] Suppose that the Leslie matrix of a population is of the form

$$L = \begin{bmatrix} 2 & 3 & 2 & 1 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}.$$

a) [3 marks] Interpret the matrix.

b) [4 marks] Determine what happens if you follow the population for 2 breeding seasons, staring with 100 zero-year old females.

12. [5 marks] Use the definition of continuity to show that

$$f(x,y)=\begin{cases}\dfrac{4xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$$

Is discontinuous at (0,0).

13. [4 marks] Suppose $f(x,y)=\sqrt{y^2-x+1}$.

a) [2 marks] Find the largest possible domain for the function and draw the sketch.

b) [2 marks] Find the equation of the level curves.

14. [6 marks] Show that the given functions are the solutions to the indicated equations:

a) [3 marks] $u(x,y) = e^x \sin y$, $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ (Laplace's equation).

b) [3 marks] $u(x,t) = \sin(x - at)$, $\dfrac{\partial^2 u}{\partial t^2} = a^2 \dfrac{\partial^2 u}{\partial x^2}$ (wave equation).