SIMON FRASER UNIVERSITY

MATH 155 FINAL EXAM

Instructor: A. Wise

August 5th, 2003, 3:30-6:30pm

Last Name ______________________

Given Name(s) ______________________

Student # ______________________

Student signature

INSTRUCTIONS

1. **Do not open this booklet until told to do so**.
2. Print your name and student number clearly, and sign your name in the space provided above.
3. For each question write your final answer in the box when one is provided. You must show all your work when space is provided. If this space is insufficient you may use the back of the previous page.
4. This exam contains this cover page, the normal distribution table, 2 formula sheets and 15 pages with a total of 14 questions. Once the exam begins please check to make sure your exam is complete.
5. No book, paper, or device, other than the usual writing instruments and this booklet, shall be within reach of a student during the examination. In particular, **no calculators are allowed.**
6. **During the examination, speaking to, communicating with, or exposing written papers to the view of, other examinees is forbidden.**
7. **Students observed writing anything after the call to stop writing will be subject to summary penalties.**

Question	Maximum	Score
1	12	
2	5	
3	5	
4	8	
5	6	
6	8	
7	7	
8	12	
9	5	
10	9	
11	5	
12	5	
13	5	
14	8	
Total	100	

1. In each case find the indefinite integral.

[3] (a) $\int 6x^2 \left(x^3+1\right)^5 \, dx$

ANSWER

SHOW YOUR WORK

[3] (b) $\int xe^{2x} \, dx$

ANSWER

SHOW YOUR WORK

[3] (c) $\displaystyle\int \frac{\csc^2 x}{\cot^3 x}\,dx$

ANSWER

SHOW YOUR WORK

[3] (d) $\displaystyle\int \frac{1}{2+3x^2}\,dx.$

ANSWER

SHOW YOUR WORK

[5] **2.** Evaluate the integral $\int_{-2}^{2}(1-|x|)\,dx$ by interpreting it in terms of areas of rectangles and/or triangles. Provide a sketch showing these areas.

ANSWER

SHOW YOUR WORK

[5] **3.** Evaluate the limit

$$\lim_{n\to\infty}\frac{\cos(3/n)+\cos(6/n)+\cos(9/n)+\ldots+\cos(3)}{n}$$

by first recognizing it as a a limit of a Riemann sum and evaluating the equivalent definite integral.

ANSWER:

SHOW YOUR WORK

4. Determine whether the improper integral diverges or converges. If it diverges explain why, and if it converges find the value that it converges to.

[4] (a) $\displaystyle\int_0^\infty \sin(2x)\,dx.$

ANSWER

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[4] (b) $\displaystyle\int_0^2 \frac{1}{x-1}\,dx.$

ANSWER

SHOW YOUR WORK

[4] **5.** (a) **Derive the formula** for the trapezoidal rule for approximating $\int_1^5 f(x)\,dx$,

$$\int_1^5 f(x)\,dx \approx \frac{1}{2}\left[f(1)+2f(2)+2f(3)+2f(4)+f(5)\right]$$

using 4 trapezoids.

The only formula you may use in your derivation is that for the area A of a trapezoid with bases p and q and height h, namely $A = \frac{h}{2}(p+q)$.

Provide a sketch showing a positive function $f(x)$ and shading the region whose area is equal to this **approximated** value.

DERIVATION and SKETCH:

[2] (b) If $f''(x) > 0$ for all x in the interval $[a,b]$, will the Trapezoidal rule yield a result greater than or less than $\int_a^b f(x)\,dx$? Explain.

ANSWER and EXPLANATION:

[4] **6.** (a) Suppose a variable X is distributed according to the function $f(x)$ defined by

$$f(x) = \begin{cases} C\dfrac{1}{(x+1)^2} & \text{if } x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of the constant C so that $f(x)$ is a probability density function.

ANSWER

SHOW YOUR WORK:

[2] (b) If X is distributed according to $f(x)$ as defined in part (a), set up the integral which needs to be evaluated in order to find the mean of X, i.e. $E(X)$. **Do not attempt to evaluate the integral**.

ANSWER

[2] (c) Set up the integral which needs to be evaluated in order to find the probability that the variable X from part (a) assumes values less than 4. **Do not attempt to evaluate the integral**.

ANSWER

7. Suppose a variable X is distributed normally with mean $\mu = -1$ and standard deviation $\sigma = 2$.

[3] (a) Find the probability that X assumes values between 0 and 1.

ANSWER

SHOW YOUR WORK:

[4] (b) Find the number m such that $P(|X - 5| > m) = 0.4$.

ANSWER

SHOW YOUR WORK

8. Consider the hierarchical competition model with two species:

$$\frac{dp_1}{dt} = 2p_1(1 - p_1) - p_1$$
$$\frac{dp_2}{dt} = 5p_2(1 - p_1 - p_2) - p_2 - 2p_1p_2$$

where $p_1(t)$ and $p_2(t)$ stand for the proportion of patches occupied by Species 1 and Species 2 respectively.

[4] (a) Find all equilibria and list them as **ordered pairs** $(\hat{p_1}, \hat{p_2})$.

ANSWER

SHOW YOUR WORK

[4] (b) If initially 0 of the patches are occupied by Species 1 and $1/6$ of the patches are occupied by Species 2 (i.e. $p_1(0) = 0$ and $p_2(0) = 1/6$), what are the values of the limits $\lim_{t\to\infty} p_1(t)$ and $\lim_{t\to\infty} p_2(t)$? Expalin.

ANSWER:

$\lim_{t\to\infty} p_1(t) =$

$\lim_{t\to\infty} p_2(t) =$

EXPLANATION:

ANSWER

[4] (c) Discuss the stability of the **nontrivial** equilibrium which you obtained in part (a) (i.e. where $\hat{p_1}$ and $\hat{p_2}$ are both **non-zero**).

SHOW YOUR WORK

9. Suppose $f(x, y) = \sqrt{y^2 - x}$.

[2] (a) Find the maximal domain D of $f(x, y)$. Find the corresponding range R.

ANSWER:
$D =$

$R =$

SHOW YOUR WORK

[3] (b) Find the equation of the level curves $c = f(x, y)$. For $c = 1$ and $c = 2$ sketch these level curves on the same two-dimensional Cartesian plane.

ANSWER

SHOW YOUR WORK

[4] **10.** (a) Show that the lilmit $\lim_{(x,y)\to(0,0)} \dfrac{x+y}{x-y}$ does not exist.

SHOW YOUR WORK

[5] (b) Show that the limit $\lim_{(x,y)\to(0,0)} \dfrac{5xy^2}{x^2+y^2} = 0.$

SHOW YOUR WORK

[1] **11.** (a) Give the definition of a function $f(x,y)$ continuous at $(x,y)=(a,b)$.

DEFINITION:

[4] (b) For what values of (x,y) is the function

$$f(x,y)=\begin{cases} \dfrac{x+y}{x-y} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$$

continuous?
Give your reasoning using the defintion of continuity.

ANSWER

SHOW YOUR WORK:

[5] **12.** Does a function $f(x, y)$ passing through the point $(1, -1, 3)$ exist with partial derivatives $f_x(x, y) = 2xy$ and $f_y(x, y) = x^2 + 3y^2$? If yes, find such a function. If no, explain why not.

SHOW YOUR WORK

[5] **13.** For the function $f(x, y, z) = xze^{yz}$ find f_x, f_y, and f_{yz} at $(1, 1, -1)$.

ANSWER:
$f_x(1, 1, -1) =$
$f_y(1, 1, -1) =$
$f_{yz}(1, 1, -1) =$

SHOW YOUR WORK

14. Suppose you are standing at $(1,-1,3)$ on a hill, the surface of which is described by the function $f(x,y)=6+2x^2y-xy^2$ (in units meters) with the positive x-axis representing South and the positive y-axis representing West.

[4] (a) If you walk **North** starting from $(1,-1,3)$ are you descending or ascending? How much is your descend/ascend for every meter of change in the North direction?

ANSWER:

SHOW YOUR WORK

[4] (b) If you walk **West** starting from $(1,-1,3)$ are you descending or ascending? How much is your descend/ascend for every meter of change in the West direction?

ANSWER:

SHOW YOUR WORK

-THE END-