SIMON FRASER UNIVERSITY

MATH 155 Final Examination

12 April 2007, 08:30–11:30

Last Name	____________________
Given Name(s)	____________________
Student #	____________________
Signature	____________________

INSTRUCTIONS

1. **Do not open this booklet until told to do so**.
2. Write your last name, given name(s), and student number in the box above. Sign on the last line of the box.
3. This exam has 12 questions on 13 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden**.
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	8	
2	8	
3	6	
4	9	
5	8	
6	6	
7	8	
8	11	
9	8	
10	9	
11	11	
12	8	
Total	100	

[2] **1.** (a) Use the substitution $u = \sin x$ to find $\int \sin x \cos x \, dx$.

[2] (b) Use the substitution $u = \cos x$ to find $\int \sin x \cos x \, dx$.

[4] (c) The antiderivatives obtained in parts (a) and (b) are different. Write down the general relation satisfied by any two antiderivatives of the same function. Verify that this relation holds for the antiderivatives obtained in parts (a) and (b) above.

[8] **2.** Find $\int e^x \sin x \, dx$.

[6] **3.** Suppose that h hours after midnight, the outside air temperature (in degrees Celsius) can be modeled by the formula

$$T(h) = 6 - \frac{1}{5}(h - 13)^2.$$

Find the average temperature between 5:00 A.M. and 2:00 P.M.

[9] **4.** An orange has the shape of a perfect sphere (ball) of radius 5 cm. Suppose that we divide the orange into two pieces by a planar cut; the radius of the circle created by the cut is 4 cm. Use an integral to determine the volume of the smaller of the two pieces (a so-called *spherical cap*) that we obtained after the cut.

[8] **5.** Evaluate $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$.

[6] **6.** Suppose that you found an approximation of $\displaystyle\int_1^2 \frac{dx}{x}$ using the Trapezoidal Rule with $n = 1,000$ intervals. Is this approximate value of the integral *larger* or *smaller* than the exact value of the integral? Explain your answer. You are *not* allowed to perform any actual numerical computations with the Trapezoidal Rule.

Hint: Draw a graph!

[8] **7.** Compute an approximation of $\ln(0.9)$ with the accuracy of three places after the decimal point. You are only allowed to use the basic arithmetic operations $+, -, \times, \div$.

Hint: Use Taylor polynomials of increasing degree. Stop as soon as two successive approximations (computed to the full precision of your calculator) agree after being rounded to the target precision. Show also the values before rounding.

8. The rate at which the temperature of an object changes is proportional to (i.e., it is a constant multiple of) the difference between its own temperature and the temperature of the surrounding medium.

[5] (a) Let $T(t)$ denote the temperature of the object at time t. Let the constant of proportionality be denoted k. Assume that the temperature of the surrounding medium is constant and denoted by M. Describe the situation given in the previous paragraph by a differential equation. Indicate whether in your model the constant k is positive or negative.

[6] (b) Assume that $T(0) = 80$, $M = 30$ and $k = 0.2$ or $k = -0.2$ depending on the choice that you made in part (a). Find $T(10)$, i.e. the temperature of the object at time $t = 10$.

[3] **9.** (a) The *Logistic Growth* model *with harvesting* is described by the differential equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H$$

where $N(t)$ is the population size at time t, and H is the constant *harvest rate.* Explain the meaning of the other constants present in the model.

[5] (b) What is the maximum harvest rate $H_{\max}$ that allows to maintain a positive population size? Hint: You need a locally stable equilibrium of positive value. Express $H_{\max}$ as a function of the other parameters introduced in part (a).

10. The *Levins model of metapopulations* is given by the differential equation

$$\frac{dp}{dt} = cp(1-p) - mp.$$

[3] (a) Explain the meaning of $p(t)$, c and m in the differential equation written above.

[6] (b) Analyze the equilibria for the differential equation written above. Remember that there are two cases to distinguish.

11. Suppose that the dynamics of a certain population is given by the following Leslie matrix:

$$L = \begin{pmatrix} 0 & 1.3 & 2.2 \\ 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \end{pmatrix}$$

At time $t = 0$ the number of females in the population is given by

$$N(0) = \begin{pmatrix} 947 \\ 473 \\ 236 \end{pmatrix}.$$

[5] (a) Compute the sizes of the age groups at the end of the next breeding season. The population is approaching a stable age distribution. What may be a possible value of λ? (Hint: λ is a "nice" number.)

[6] (b) The question **11.** continues here.

Using the guess that you made in regards to λ in part (a), set up a system of linear equations whose solution is a stable age distribution. Find the exact pattern of the stable age distribution.

12. The function

$$f(x, y) = x + 2y - xy^2$$

is defined for all pairs $(x, y) \in \mathbb{R}^2$.

[4] (a) Find all critical points for f.

[4] (b) For each critical point found in part (a), determine its nature.