

MATH 155

FINAL

9:00-12:00, April 14, 2004

Instructor: Peter Berg

Family name: _____

Initials: _____

Student ID number: _____

Signature: _____

READ INSTRUCTIONS CAREFULLY:

- **Do not lift the cover page until instructed!**
- Fill out your name and ID in the space provided and sign.
- You **MUST NOT** use a calculator. NO other aids.
- Answer all questions, explaining your answers carefully in the space provided. If you run out of space, use the back of the **preceding** page.
- This exam consists of 10 questions and 11 pages (including this one).

Question	1	2	3	4	5	
Grade	/10	/10	/10	/10	/10	
	6	7	8	9	10	Total
	/10	/10	/10	/10	/10	/100

Good Luck!

1. (10 marks) *Battle of Orders: Logarithm vs Polynomial*

Evaluate

$$\int_1^{\infty} \frac{\ln x}{x^2} dx$$

and sketch the graph of the integrand over the range of integration.

2. (10 marks) *Slicing through Bert's head*

The straight line $y = 1 - x$ breaks the interior of the ellipse

$$2x^2 + y^2 = 1$$

into two parts. What is the size of the smaller area?

3. (10 marks) *AWOL in BC*

The population dynamics of a little gulf island is described by

$$\frac{dN}{dt} = tN + 4t$$

where $N(0) = 100$. What is $N(t)$ for $t \geq 0$? What is the population size after $t = 1$ (year)?

4. (10 marks) *3 times 3 times 3 times 3*

Let us consider the following matrices

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & -3 & 1 \\ 2 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 0 & 3 \\ 0 & 2 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, C = \begin{pmatrix} 7 \\ -5 \\ 7 \end{pmatrix}.$$

Compute BA and solve $AX = C$.

5. (10 marks) *Think positive and negative and positive.*

What is the size of the area between $x = 0$, $x = 3$, $y = 0$ and the function

$$f(x) = x^2 - 3x + 2 \quad ?$$

6. (10 marks) *C'mon!*

i) What is

$$\frac{d^2}{dx^2} \int \sin(x^2) dx \quad ?$$

ii) What is

$$\int \frac{d^2}{dx^2} (\sin x)^2 dx \quad ?$$

7. (10 marks) *The infinite tear drop*

We look from the side onto an infinite tear drop with a cross-sectional area described by

$$f(x) = \pm \frac{\sqrt{x}}{x+1} .$$

What is the volume of the tear drop whose cross-sectional area is circular, when we slice through it perpendicular to the x-axis? What is the radius of the tear drop when it assumes a spherical shape?

8. (10 marks) *Positive, negative or zero?*

Sketch the graph of the function

$$f(x) = \exp(-x) \sin(x)$$

and compute its integral for $x \in [0; \infty)$.

9. (10 marks) *Two dead seas, one sea level and two endless walls to climb.*

Find the equilibria of the differential equation

$$\frac{dy}{dx} = y^4 - 3y^3 + 3y^2.$$

What is the linear stability of these equilibria? (Use eigenvalues!)

Graph $\frac{dy}{dt}$ as a function of y and discuss global stability.

10. (10 marks) *Turning logarithms into polynomials*

Determine the Taylor polynomial of order 5, $P_5(x)$, of the function $y(x) = \ln(x)$ about $x = 1$. What is $y(e)$? Is $P_5(e)$ less or greater than $y(e)$?