BLUE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 2

17 March 2010, 08:30–09:20

Last Name ____________________

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INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Fill out the box in the upper right corner of this page.
3. This exam has 6 questions on 6 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.**
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	7	
2	8	
3	7	
4	4	
5	7	
6	7	
Total	40	

[7] **1.** Find $\int \frac{x+11}{x^2+4x-5}\,dx$.

$$x^2+4x-5=0$$

$$x=\frac{-4\pm\sqrt{16+20}}{2}=\begin{cases}1\\-5\end{cases}$$

$$\frac{x+11}{x^2+4x-5}=\frac{A}{x-1}+\frac{B}{x+5}=\frac{A(x+5)+B(x-1)}{(x-1)(x+5)}=$$

$$=\frac{(A+B)x+5A-B}{(x-1)(x+5)}$$

$$\left.\begin{array}{l}A+B=1\\5A-B=11\end{array}\right\}\Rightarrow\begin{array}{l}B=1-A\\5A-(1-A)=11\\A=2,\ B=-1\end{array}$$

$$\int\frac{x+11}{x^2+4x-5}\,dx=\int\frac{2}{x-1}\,dx+\int\frac{-1}{x+5}\,dx=$$

$$=2\ln|x-1|-\ln|x+5|+C$$

2. For each of the following two improper integrals, determine whether it is convergent or divergent. If the integral is convergent, determine its value.

[4] (a) $\int_1^4 \frac{2}{(x-3)^2}\,dx$

$$\int_1^4 \frac{2}{(x-3)^2}\,dx = \int_1^3 \frac{2}{(x-3)^2}\,dx + \int_3^4 \frac{2}{(x-3)^2}\,dx =$$

$$= \lim_{c\to 3^-} \int_1^c \frac{2}{(x-3)^2}\,dx + \lim_{c\to 3^+} \int_c^4 \frac{2}{(x-3)^2}\,dx = \quad \boxed{\begin{array}{l} u = x-3 \\ du = dx \end{array}}$$

$$= \underbrace{\lim_{c\to 3^-} \left[-\frac{2}{x-3}\right]_1^c}_{\infty} + \underbrace{\lim_{c\to 3^+} \left[-\frac{2}{x-3}\right]_c^4}_{\infty}$$

The integral is divergent.

[4] (b) $\int_5^\infty \frac{2}{(x-3)^2}\,dx$

$$\int_5^\infty \frac{2}{(x-3)^2}\,dx = \lim_{c\to\infty} \int_5^c \frac{2}{(x-3)^2}\,dx =$$

$$= \lim_{c\to\infty} \left[-\frac{2}{x-3}\right]_5^c = \lim_{c\to\infty} -\frac{2}{c-3} - \left(-\frac{2}{5-3}\right) =$$

$$= 0 + 1 = 1$$

[7] **3.** Evaluate $\int x^2 \cos x\, dx$.

$$\boxed{u = x^2, \quad v' = \cos x, \quad v = \sin x}$$

$$\int x^2 \cos x\, dx = x^2 \sin x - \int 2x \sin x\, dx =$$

$$\boxed{u = 2x, \quad v' = \sin x, \quad v = -\cos x}$$

$$= x^2 \sin x - \left(2x(-\cos x) - \int 2(-\cos x)\, dx\right) =$$

$$= x^2 \sin x + 2x \cos x - \int 2 \cos x\, dx =$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

[4] **4.** Use the trapezoidal rule with $n = 4$ intervals to approximate the value of $\int_3^5 x^2\,dx$. Compare the approximation with the exact value of the integral.

$$\Delta x = \frac{5-3}{4} = 0.5$$

$$x_0 = 3, \quad x_1 = 3.5, \quad x_2 = 4, \quad x_3 = 4.5, \quad x_4 = 5$$

$$\int_3^5 x^2\,dx \approx \frac{5-3}{4}\left(\frac{3^2}{2} + 3.5^2 + 4^2 + 4.5^2 + \frac{5^2}{2}\right) =$$

$$= 32.75$$

$$\int_3^5 x^2\,dx = \left[\frac{x^3}{3}\right]_3^5 = \frac{125}{3} - \frac{27}{3} =$$

$$= \frac{98}{3} \approx 32.67$$

[7] **5.** Solve the differential equation

$$\frac{dy}{dx} = 2\sqrt{y}$$

with the initial condition $y(0) = 4$.

$$\frac{dy}{dx} = 2\sqrt{y}$$

$$\frac{dy}{2\sqrt{y}} = dx$$

$$\int \frac{dy}{2\sqrt{y}} = \int dx$$

$$\sqrt{y} = x + C$$

$$y(0) = 4 \Rightarrow \sqrt{4} = 0 + C \Rightarrow C = 2$$

$$\sqrt{y} = x + 2$$

$$y(x) = (x+2)^2$$

6. Let r and K be positive constants. The differential equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

describes a well known growth model.

[3] (a) Give the name of the model and describe the meaning of the symbols in the differential equation.

Logistic Equation (or Logistic Growth Model)

N... population size

t ... time

r ... intrinsic rate of growth

K ... carrying capacity

[4] (b) Let $r = 0.1$ and $K = 100$. Find the equilibria of the differential equation and the eigenvalues associated with them. Use the eigenvalues to determine the stability of the equilibria.

$g(N) = 0.1\, N\left(1 - \frac{N}{100}\right)$

equilibria: $g(\hat{N}) = 0 \Rightarrow \hat{N} = 0$ or $\hat{N} = 100$

$g'(N) = 0.1\left(1 - \frac{N}{100}\right) + 0.1\,N\left(-\frac{1}{100}\right)$

$g'(0) = 0.1 \Rightarrow \hat{N} = 0$ is an unstable equilibrium

$g'(100) = 0.1 \cdot 100 \cdot \left(-\frac{1}{100}\right) = -0.1$

$\Rightarrow \hat{N} = 100$ is a locally stable equilibrium