BLUE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 1

3 February 2010, 08:30–09:20

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INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Fill out the box in the upper right corner of this page.
3. This exam has 6 questions on 5 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.**
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	7	
2	4	
3	5	
4	8	
5	8	
6	8	
Total	40	

1. Indicate whether the following statements are true (T) or false (F). Justifications are **not required.**

Assume that $f(x)$ is continuous in the intervals of integration.

A statement containing general constants a, b, c and functions f, g is true if and only if it holds for *all admissible choices* that you can make for a, b, c, f, g.

[1] (a) __T__ If $f(x) \geq c$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq c(b-a)$.

[1] (b) __T__ $\int_a^b (f(x) - g(x))\,dx = \int_a^b f(x)\,dx + \int_b^a g(x)\,dx$

[1] (c) __F__ $\int_a^b (f(x))^2\,dx = \left(\int_a^b f(x)\,dx\right)^2$ consider $f(x)=1$

[1] (d) __F__ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx \cancel{-} \int_c^b f(x)\,dx$ $+$

[1] (e) __F__ $\sum_{k=10}^{20} f(k) = \sum_{k=1}^{20} f(k) - \sum_{k=1}^{10} f(k)$ $LHS = \sum_{k=11}^{20} f(k)$

[1] (f) __T__ $\int_{-a}^{a} 3\sin x\,dx = 0$

[1] (g) __F__ There is a point d in $[a, b]$ such that $f(d) = \int_a^b f(x)\,dx$.

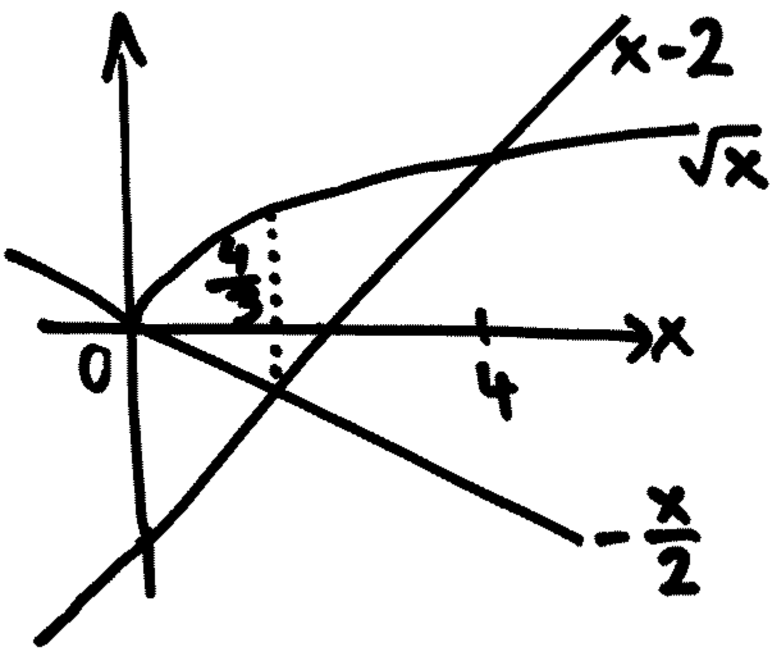
$f(d) \cdot (b-a) = \ldots$

[4] **2.** Determine the average value of $f(x) = 4x^3 + 5$ in the interval $[1, 4]$.

$$f_{avg} = \frac{1}{4-1}\int_1^4 4x^3+5\,dx =$$

$$= \frac{1}{3}\left[x^4+5x\right]_1^4 = \frac{1}{3}(276-6) = 90$$

[5] **3.** Graph the finite region that is enclosed by the curves $y = \sqrt{x}$, $y = -\frac{x}{2}$ and $y = x - 2$. Express the area of this region as the sum of two definite integrals. **DO NOT EVALUATE THE INTEGRALS.**



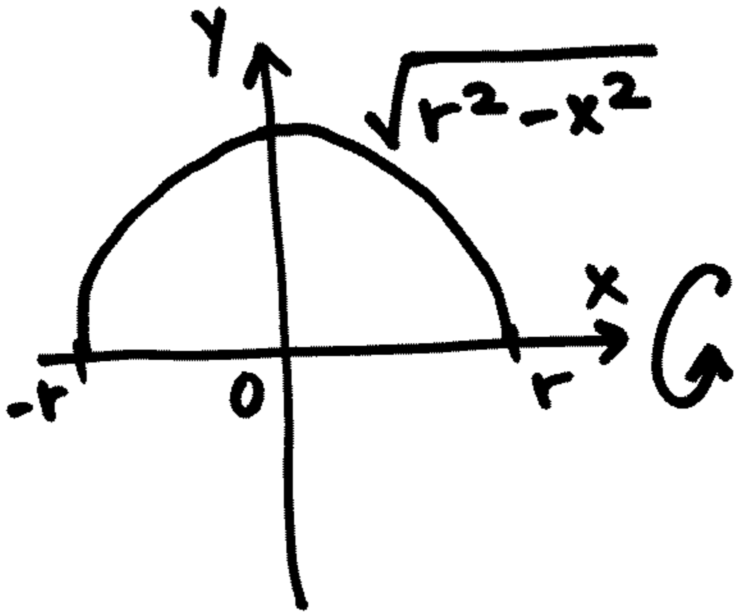
$$x-2 = \sqrt{x}$$
$$\Downarrow$$
$$x=4, \cancel{x=1}$$

$$-\frac{x}{2} = \sqrt{x}$$
$$\Downarrow$$
$$x=0, \cancel{x=4}$$

$$x-2 = -\frac{x}{2} \Rightarrow x = \frac{4}{3}$$

$$\text{Area} = \int_0^{4/3}\left(\sqrt{x}-\left(-\frac{x}{2}\right)\right)dx + \int_{4/3}^{4}\left(\sqrt{x}-(x-2)\right)dx$$

[4] **4.** (a) Express the sphere of radius r as a solid of revolution. (This involves specifying a certain function and an interval on the x-axis.)



(x,y) on semicircle

$$\Downarrow$$

$$x^2+y^2=r^2$$

$$\Downarrow$$

$$y=\sqrt{r^2-x^2}$$

[4] (b) Use the formula for the volume of a solid of revolution to evaluate the volume of the sphere of radius r.

$$\text{Volume} = \int_{-r}^{r} \pi\left(\sqrt{r^2-x^2}\right)^2 dx =$$

$$= \pi\int_{-r}^{r}(r^2-x^2)\,dx = \pi\left[r^2x-\frac{x^3}{3}\right]_{-r}^{r} =$$

$$= \pi\left(r^3-\frac{r^3}{3}-\left(-r^3+\frac{r^3}{3}\right)\right) = \frac{4}{3}\pi r^3$$

[8] 5. Find $\displaystyle\int \frac{(\sin x)^3}{\cos x}\,dx$.

Hint: Split the numerator into two factors, and use $(\sin x)^2 + (\cos x)^2 = 1$.

$$\int \frac{(\sin x)^3}{\cos x}\,dx = \int \frac{(\sin x)^2 \cdot \sin x}{\cos x}\,dx =$$

$$= \int \frac{\left(1 - (\cos x)^2\right) \cdot \sin x}{\cos x}\,dx =$$

$$\boxed{\begin{aligned} u &= \cos x \\ \frac{du}{dx} &= -\sin x \\ du &= -\sin x\,dx \end{aligned}}$$

$$= \int \frac{(1-u^2)(-du)}{u} =$$

$$= \int \frac{-du}{u} + \int u\,du =$$

$$= -\ln|u| + \frac{u^2}{2} + C =$$

$$= -\ln|\cos x| + \frac{(\cos x)^2}{2} + C$$

[8] **6.** Evaluate $\int_3^4 \frac{5}{x\sqrt{\ln(3x)}}\,dx$.

$$\boxed{\begin{aligned} u &= \ln(3x) \\ \frac{du}{dx} &= \frac{1}{3x}\cdot 3 \\ du &= \frac{dx}{x} \end{aligned}}$$

$$\int_3^4 \frac{5}{x\sqrt{\ln(3x)}}\,dx = \int_{\ln 9}^{\ln 12} \frac{5\,du}{\sqrt{u}} =$$

$$= 5\int_{\ln 9}^{\ln 12} u^{-1/2}\,du = 5\left[\frac{u^{1/2}}{\frac{1}{2}}\right]_{\ln 9}^{\ln 12} =$$

$$= 5\left(2\sqrt{\ln 12} - 2\sqrt{\ln 9}\right) \approx$$

$$\approx 0.9405$$