Simon Fraser University

MATH 154, Spring 2010

Midterm 2

March 17, 2010, 8:30 - 9:20

Last Name (print)	
First Name (print)	
SFU ID	
SFU email	@sfu.ca

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 5 questions. Once the exam begins please check to make sure your exam is complete.
4. FOR FULL MARK YOU MUST SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Question	Marks
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

1. **[10 marks]** Answer T (true) or F (false) in the boxes provided or leave the box blank. No explanation is necessary. For every correct answer you receive 2, for an incorrect answer you receive -1. You may leave the box empty in which case you receive 0.

a) $\boxed{\text{T}}$ Let f be continuous on $[a, b]$ and let $f(a) < L < f(b)$ for some $L \in \mathbb{R}$. Then there exists a constant $c \in (a, b)$ such that $f(c) = L$.

b) $\boxed{\text{F}}$ If f is continuous at x, then it is differentiable at x.

c) $\boxed{\text{F}}$ If the line $x = c$ is a vertical asymptote of f, then EITHER $\lim_{x \to c^-} f(x) = +\infty$ and $\lim_{x \to c^+} f(x) = +\infty$ OR $\lim_{x \to c^-} f(x) = -\infty$ and $\lim_{x \to c^+} f(x) = -\infty$.

d) $\boxed{\text{T}}$ The derivative of a rational function is a rational function.

e) $\boxed{\text{F}}$ Given function f and a constant $h \neq 0$. The quotient $\frac{f(x+h)-f(x)}{h}$ is the derivative of f at point x.

2. **[5 marks each]**

a) In two cities recorded day temperatures were given by $f(t) = -12\sin(\frac{\pi}{12}t - \frac{3\pi}{2}) + 8$ and $g(t) = -8\sin(\frac{\pi}{12}t - \frac{3\pi}{2}) + 8$, respectively. ($t$ is the time of the day, i.e. $t \in [0, 24)$). Show that there was a time before noon at which the temperatures in both cities were the same. (Time before noon is from 0 to 12 hours.)

- f, g continuous on $[0,12]$ $\Rightarrow$ $h(t) = f(t) - g(t)$ continuous on $[0,12]$
- $h(0) = f(0) - g(0) = -12 \cdot \sin(-\frac{3\pi}{2}) + 8 - (-8 \cdot \sin(-\frac{3\pi}{2}) + 8) = -12 + 8 - (-8 + 8) = -4$
- $h(12) = f(12) - g(12) = -12 \cdot \sin(-\frac{\pi}{2}) + 8 - (-8 \cdot \sin(-\frac{\pi}{2}) + 8) = 20 - 16 = 4$
- $h(0) < 0 < h(12)$
- I.V.T. $\Rightarrow$ $\exists\, \ell \in (0, 12)$: $h(\ell) = f(\ell) - g(\ell) = 0$ $\Rightarrow$ $\underline{f(\ell) = g(\ell)}$.

At time ℓ the temperature were the same in both cities.

b) How many steps in the Bisection Method are required to guarantee that an approximate root of $x^5 - 7x + 3 = 0$ is within 0.0001 of the true value of the root? Suppose we start on interval [-1,2] and in each iteration we halve the selected sub-interval.

- original interval length 3
- each iteration length halves
- final length (after r iterations) must be $< 10^{-4}$

$$3 \cdot \left(\frac{1}{2}\right)^r < 10^{-4}$$

$$\log 3 + r \cdot \log \frac{1}{2} < -4$$

$$r > \frac{-4 - \log 3}{\log \frac{1}{2}} \simeq 14.9$$

Hence 14 iterations are not enough but 15 are enough.

3. **[5 marks each]**

a) From the definition of the derivative compute $f'(x)$ where $f(x) = \frac{1}{x+2}$ for $x \neq -2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{x+2-(x+2+h)}{(x+h+2)(x+2)}}{h} = \lim_{h\to 0} -\frac{1}{(x+2)(x+2+h)} = -\frac{1}{(x+2)^2}.$$

b) Let $f(x) = x^{\frac{1}{5}}$. Compute $f'(0)$ or show (from definition) it does not exist, i.e. $f(x)$ is not differentiable at $x = 0$.

for $x = 0$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^{1/5}}{h} = \lim_{h\to 0} \frac{1}{h^{4/5}} = \infty$$

limit does not exist, hence $f(x)$ is not differentiable at $x = 0$.

4. Compute the derivative of the following functions. You may use any method. Do not simplify your answer.

a) **[3 marks]** Given $f(x) = (1-(3x^3-x)^3)^3$. Compute $f'(x)$.

$$\frac{d}{dx}\left[(1-(3x^3-x)^3)^3\right] = 3(1-(3x^3-x)^3)^2 \cdot \frac{d}{dx}\left[1-(3x^3-x)^3\right] =$$

$$= \underline{3(1-(3x^3-x)^3)^2 \cdot (-3)(3x^3-x)^2(9x^2-1)}.$$

$$\frac{d}{dx}\left[1-(3x^3-x)^3\right] = \frac{d}{dx}\left[-(3x^3-x)^3\right] = -3(3x^3-x)^2 \cdot \frac{d}{dx}\left[3x^3-x\right] =$$

$$= \underline{-3(3x^3-x)^2(9x^2-1)}$$

$$\frac{d}{dx}\left[3x^3-x\right] = \underline{9x^2-1}$$

b) **[3 marks]** Given $f(x) = \frac{3x-1}{\sqrt{2x^2+1}}$. Compute $f'(x)$.

$$\frac{d}{dx}\left[\frac{3x-1}{\sqrt{2x^2+1}}\right] = \frac{3\cdot\sqrt{2x^2+1} - (3x-1)\cdot\frac{d}{dx}\left[\sqrt{2x^2+1}\right]}{2x^2+1} = \underline{\underline{\frac{3}{\sqrt{2x^2+1}} - \frac{(3x-1)\cdot 4x}{2(2x^2+1)^{3/2}}}}.$$

$$\frac{d}{dx}\left[\sqrt{2x^2+1}\right] = \frac{4x}{2\sqrt{2x^2+1}}.$$

c) **[4 marks]** Suppose $f'(x) = \frac{1}{x+1}$. Compute $f'(\sqrt{x^2+1})$.

let $u = \sqrt{x^2+1}$

$$\frac{d}{dx} f(\sqrt{x^2+1}) = \frac{d}{dx} f(u) = f'(u)\cdot u' = \frac{1}{u+1}\cdot\frac{2x}{2\sqrt{x^2+1}} =$$

$$= \underline{\underline{\frac{1}{\sqrt{x^2+1}+1}\cdot\frac{x}{\sqrt{x^2+1}}}}.$$

5. **[5 marks each]**

a) Find the equations of two lines that are tangent to the graph of the function $y = \frac{1}{x}$ at points $(1,1)$ and $(-1,-1)$, respectively.

$y' = -\frac{1}{x^2}$

at $(1,1)$ $y' = -1$ - slope

tangent line : $(y-1) = -1(x-1)$

$\underline{y = 2 - x}$;

at $(-1,-1)$ $y' = -1$ - slope

tangent line : $(y+1) = -1(x+1)$

$\underline{y = -2 - x}$;

b) The growth of a population of a bacteria at time t satisfies $N(t) = 10^6 - (3(t-1)^2 - 6t + 6)^2$. What is the instantaneous population growth rate at time t? Determine all times at which this instantaneous growth rate will be zero, i.e. times at which the population growth changes character from increasing to decreasing or vice versa.

- instantaneous population growth:

$$\frac{dN}{dt} = -2(3(t-1)^2 - 6t + 6)\cdot(6(t-1) - 6) =$$

$$= \underline{\underline{-2(3(t-1)^2 - 6t + 6)\cdot(6t - 12)}}.$$

- $-2(3(t-1)^2 - 6t + 6)\cdot(6t-12) = 0$

$6t - 12 = 0$

$\underline{\underline{t = 2}}$;

or

$3t^2 - 6t + 3 - 6t + 6 = 0$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$\underline{\underline{t = 1}}$; or $\underline{\underline{t = 3}}$;