Blue Solutions

Math 154, Calculus I for the Biological Sciences

Midterm II, Mar. 5

Instructor: Matt DeVos

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1. DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
2. Fill in the above box.
3. Once the exam begins, check that your exam has 6 questions and 7 pages.
4. Only the usual writing instruments, this booklet, and a scientific calculators are allowed. **No graphing or programmable calculators are permitted.**
5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.
6. You may use the back of the previous page for rough work or if you run out of space.
7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Problem	Score	Value
1		8
2		8
3		9
4		10
5		7
6		8
Total:		50

Problem 1. *(8 points)* Mark each statement as true (**T**) or false (**F**)

F If f is continuous at x, then f is differentiable at x.

F If $f(x) = \tan x$, then $f'(x) = \csc^2 x$.

F If $f(1) = 2$ and $f'(1) = 3$, the linearization of f at 1 is $L(x) = 1 + 3(x - 2)$.

T The derivative of a rational function is a rational function.

Problem 2. *(8 points)*

(3 points) State the definition of the derivative of a function f at x.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(5 points) Use the definition of the derivative to compute $f'(x)$ when $f(x) = \sqrt{x} + 4$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} + 4 - (\sqrt{x} + 4)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x + h - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\underbrace{\sqrt{x+h}}_{\sqrt{x}} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Problem 3. *(9 points)* Compute the derivatives of the following functions (you may use any method from class, and you need not simplify your answers).

(3 points)

$$f(x) = \ln(x^3 + x)$$

$$f'(x) = \frac{3x^2+1}{x^3}$$

(3 points)

$$f(x) = \frac{3e^{x^2}}{\cos^2 x}$$

$$f'(x) = \frac{3e^{x^2} \cdot 2x \cdot \cos^2 x - 3e^{x^2} \cdot 2\cos x(-\sin x)}{\cos^4 x}$$

$$= \frac{6xe^{x^2} \cdot \cos^2 x + 6e^{x^2} \cdot \cos x \cdot \sin x}{\cos^4 x}$$

(3 points)

$$y = f(x) = (\cos x)^x$$

$$\ln y = x \cdot \ln(\cos x)$$

$$\frac{y'}{y} = \ln(\cos x) + x \cdot \frac{-\sin x}{\cos x}$$

$$y' = (\cos x)^x \left(\ln(\cos x) + x \frac{-\sin x}{\cos x} \right)$$

Problem 4. *(10 points)*

(5 points) Consider the curve given by the equation $x^2y^2 - xy = 2$. Find the line tangent to this curve at the point $(2, 1)$

$$x^2y^2 - xy = 2$$

$$2xy^2 + x^2 \cdot 2y \cdot y' - (y + xy') = 0$$

$$y'(2x^2y - x) = y - 2xy^2$$

$$y' = \frac{y - 2xy^2}{2x^2y - x}$$

$$\boxed{y - 1 = -\tfrac{1}{2}(x-2)}$$

at $x=2$ $y=1$ $\quad y' = \frac{1-4}{8-2} = \frac{-3}{6} = -\frac{1}{2}$

(5 points) Find the line tangent to the graph of the function $y = 4x^2$ at the point corresponding to $x = a$.

$y = f(x) = 4x^2$

slope $= f'(a)$

$f'(x) = 8x$ so $f'(a) = 8a$

then $(a, f(a)) = (a, 4a^2)$

so $\quad y - 4a^2 = 8a(x - a)$

Problem 5. *(7 points)* A particle has position $s(t) = e^{t^3-6t^2+12t}$ at time t.

(2 points) What is the velocity at time t?

$$s'(t) = e^{t^3-6t^2+12t}\left(3t^2-12t+12\right)$$

(2 points) What is the acceleration at time t?

$$s''(t) = e^{t^3-6t^2+12t}\left(3t^2-12t+12\right)^2 + e^{t^3-6t+12t}\left(6t-12\right)$$

(3 points) Find all times t so that the particle is stopped at time t.

stopped : $s'(t) = 0$

so $3t^2-12t+12 = 0$ (since $e^x \neq 0$ for every x)

$t^2-4t+4=0$

$(t-2)^2 = 0$

$t = 2$

Problem 6. *(8 points)* A cube is expanding so that the side length is increasing at a constant rate of 3in/sec. At what rate is the volume increasing when the side length is 12in.?

① side length : $s(t)$

volume : $V(t)$

② $V = s^3$

③ $$\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt}$$

④ when $s = 12$

$$\frac{dV}{dt} = 3 \cdot 12^2 \cdot 3 = 1296 \text{ in}^3/\text{s}$$