MATH 154 Calculus I for the Biological Sciences
Midterm Exam 2

Professor: Robert Russell
Date: October 31, 2007
Time: 8:30 - 9:20am

Last Name: _____

First Name: _____

Student Number: _____

Signature: _____

- DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
- Write your name and student number on the front page.
- This exam is closed book and closed notes.
- **No graphing or programmable calculators are permitted.**
- You may lose marks if your explanations are incomplete or poorly presented.
- You may use the reverse side of the previous page for rough work or if you run out of space.
- During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.
- Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.
- This exam has 6 pages, including this cover page. Please make sure that you have all of the 6 pages.

Question	**1**	**2**	**3**	**4**	**5**	**Total**
Marks	**/20**	**/30**	**/30**	**/10**	**/15**	**/105**

Problem 1 (20pt=10+10)
Using the **formal definition of the derivative**, find

(i) $f'(2)$ for $f(x) = x^3$

and

(ii) $f'(x)$ for $f(x) = \sin(x)$. (Hint: $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$)

Problem 2 (30pt=10+10+10)
Differentiate the following functions. You do NOT need to simplify your answers.

(i) $f(x) = \sin(\frac{\pi}{8})\cos^3(\sqrt{x^2-5})$,

(ii) $h(t) = (3t^3 - \frac{4}{t^5})^8$,

(iii) $f(R) = \frac{cR}{k+R}e^{R^2}$, where c and k are constants.

Problem 3 (30pt=15+15)
(*i*) Find the tangent line to the curve $y = e^{2x}$ at the point on the curve corresponding to the value $x = a$. Find all values of a such that this tangent line passes through the point $(0, 0)$. (Note: The point $(0, 0)$ is not on the curve.)

(*ii*) Using **implicit differentiation**, find the slope of the normal line (not the tangent line) to the curve

$$y^3 - xy = -6$$

at the point $(11, 3)$.

Problem 4 (15pt)
What is the Chain Rule for evaluating $\frac{d}{dx}(f \circ g(x)) = f(g(x))$, where f and g are both differentiable functions? When $g = f^{-1}$, $f(g(x)) = x$. Using this and the formula for the derivative of the exponential function e^y, find the derivative of $\ln(x)$.

Problem 5 (15pt)
Suppose that two chemicals occurring in the atmosphere change with time in such a way that their concentrations, x and y, satisfy the relationship

$$x^3 + xy = 2y^2.$$

If $\frac{dx}{dt} = 3$ when $x = 1$, find $\frac{dy}{dt}$? (Hint: Use related rates.)