

**Simon Fraser University
Department of Mathematics
Math 154-3, Calculus I for Biological Sciences
Test 2**

Instructor: E. Lee

30 Oct 2002

Last Name

Student Number

Given Names

1. Do not turn over this cover page until instructed to do so.
2. No calculators allowed.
3. Full marks will be awarded for correct, complete and well-organized solutions. Except for Question 1, full justification of answers is required, and requested methods must be used.
4. There are 4 questions in this test. You have 50 minutes to complete this test.

For examiner's use only

Question	Marks
1	/8
2	/4
3	/5
4	/3
Total	/20

1. [8 marks] Multiple choice questions. No justification is required.

(a) Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - e^x}{\sin x}.$$

- (A) 1
- (B) -1
- (C) $\sin e$
- (D) 0
- (E) ∞

Answer

(b) Evaluate

$$\lim_{x \rightarrow \infty} \frac{|-x| + |3 - x|}{x}.$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) ∞

Answer

(c) Let

$$f(x) = \pi \sin(\sqrt{2}\pi \cos(\pi x^2)).$$

Evaluate $f'(1/2)$.

- (A) $-\pi$
- (B) $\sqrt{2}\pi$
- (C) π^2
- (D) π^3
- (E) $2\pi^3$

Answer

(d) Let

$$f(x) = 2x - \cos(-x), \quad x \text{ in } \mathbf{R}.$$

Find $\left(\frac{d}{dx}f^{-1}\right)(\pi)$. (Note that $f(\pi/2) = \pi$).

- (A) $1/\pi$
- (B) $2/\pi$
- (C) 1
- (D) $1/2$
- (E) $1/3$

Answer

- (e) For which x is the function

$$g(x) = \begin{cases} |1 - x^2| & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

not differentiable?

- (A) $x = -1$
(B) $x = 1$
(C) $x = 0$
(D) $x = -1, 0$
(E) $x = -1, 1$

Answer

- (f) Let

$$h(x) = \frac{e^{x^2}}{e^x}.$$

Evaluate $h'(1)$.

- (A) e^2
(B) e
(C) 1
(D) 2
(E) 3

Answer

(g) Let

$$f(x) = x^4 + \sin(\pi x).$$

Evaluate $f^{(3)}(-1/2)$.

- (A) -12
- (B) $-12 - \pi^3$
- (C) $-12 + \pi^3$
- (D) $3 - \pi^2$
- (E) $-1/2 + \pi$

Answer

(h) Let $W(t)$ denotes the amount of a radioactive material left after time t . Assume that $W(0) = 16$ and $W(1) = 12$. How much material is left at time $t = 3$?

- (A) 4
- (B) 8
- (C) $9/4$
- (D) $16/9$
- (E) $27/4$

Answer

2. [4 marks] Let P be a point on the graph

$$y = xe^x \quad \text{for } x > 0.$$

Suppose the x -coordinate of P increases at a constant rate of 2.5 units per second. How fast is the angle θ of inclination of the line joining P to the origin changing when the x -coordinate of P is at $x = \ln 2$ units?

3.(a) [2 marks] Find $\frac{dy}{dx}$ if $y = \sqrt{2x + \sqrt{x}}$.

(b) [3 marks] Suppose $\ln y = x^2 - \cos(xy)$. Find $\frac{dy}{dx}$ by implicit differentiation.

4. [3 marks] Find the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 27$, and use it to find the approximate value of $\sqrt[3]{28}$. You may express your answer in fractions.

Some basic information

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{h \rightarrow 0} \frac{a^h - 1}{h} = \ln a$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} a^x = (\ln a) a^x$$

$$\frac{d}{dx} \log_a x = \frac{1}{(\ln a)x}$$

$$\pi \text{ radians} = 180^\circ$$

θ	$\sin \theta$	$\cos \theta$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0