Simon Fraser University

MATH 154, Spring 2010

Midterm 1

February 03, 2010, 8:30 - 9:20

Last Name (print)	
First Name (print)	
SFU ID	
SFU email	@sfu.ca

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 5 questions. Once the exam begins please check to make sure your exam is complete.
4. FOR FULL MARK YOU MUST SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Question	Marks
1	
2	
3	
4	
5	
Total	

F

1. Answer T (true) or F (false) in the boxes provided or leave the box blank. No explanation is necessary. For every correct answer you receive 2, for an incorrect answer you receive -1. You may leave the box empty in which case you receive 0. **[12 marks]**

a) $\boxed{F}$ The functions $f(x) = \frac{x^2-1}{x-1}$, $x \neq 1$ and $g(x) = x + 1$, $x \in \mathbb{R}$ are equal.

domains not the same

b) $\boxed{T}$ The graph of function $y = (x-4)^2$ is a horizontal translation of the graph of $y = x^2$ by 4 units to the right.

c) $\boxed{T}$ The function $f(x) = \sin(x) \cdot \cos(x)$ is odd.

d) $\boxed{F}$ Let a be a constant and suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists.

may not be true if $\lim_{x \to a} g(x) = 0$

e) $\boxed{T}$ The function $f(x) = \frac{\sin(x)}{x}$ is continuous on $(0, 1)$.

f) $\boxed{F}$ The graph of a power function on a semi-log (log-linear) plot forms a straight line.

must be log-log plot

2. **[6 marks each]**

$g \circ f$

a) Given $f(x) = \ln(x-10)$ and $g(x) = \sqrt[3]{x+1}$. Find largest possible domains for $f(x)$, $g(x)$, and $(f \circ g)(x)$.

3 +

1 1 1

dom $f(x) =$ $(10, \infty)$ dom $g(x) =$ $\mathbb{R}$ dom $(g \circ f)(x) =$ $(10, \infty)$

$x - 10 > 0$

$x > 10$

rng $f \subseteq$ dom g

b) Solve for x, $\log_3 x^2 - \log_3 2x = 1$. $\longrightarrow$ $\log_3 \frac{x^2}{2x} = 1$ 1

3 +

$\left.\begin{array}{l} x^2 > 0 \\ 2x > 0 \end{array}\right\} x > 0$

1

$\frac{x^2}{2x} = 3$ 1

$x^2 = 6x$

$x = 6$

3. **[4 marks each]** Find the following limits.

2+ a) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{2x} = \lim_{x\to 0}\left[\frac{\sqrt{x^2+4}-2}{2x}\cdot\frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}\right] = \lim_{x\to 0}\frac{x^2}{2x\left(\sqrt{x^2+4}+2\right)} =$ 1

$= \lim_{x\to 0}\frac{x \to 0}{\underbrace{2\left(\sqrt{x^2+4}+2\right)}_{\to 8}} = \frac{0}{8} = \underline{\underline{0}};$ 1

2+ b) $\lim_{x\to -\infty} \frac{4-x^2}{1+3x^2} = \lim_{x\to -\infty}\frac{\frac{4}{x^2}-1}{\frac{1}{x^2}+3} = -\underline{\underline{\frac{1}{3}}};$ 1 1

$\frac{4}{x^2}\to 0$, $\frac{1}{x^2}\to 0$

2+ c) $\lim_{x\to 1^-} \frac{1-x^2}{|x-1|} = \lim_{x\to 1^-}\frac{(1-x)(1+x)}{1-x} = \lim_{x\to 1^-}(1+x) = \underline{\underline{2}};$ 1 1

$(-x-1)(x-1)$

$-(x-1)$

4. **[6 marks each]**

a) If the graph of a function $y = f(x)$ on a double-log (log-log) plot is a straight line with slope 2 and passing through $(0, 3)$, find the relationship between y and x, i.e. find the function $y = f(x)$.

$$\log y = 2 \cdot \log x + 3 \quad (1)$$

3+

$$y = 10^{2 \cdot \log x + 3} \quad (1)$$

$$\boxed{y = 10^3 \cdot x^2} \quad (1)$$

b) Suppose that a pathogen is introduced into a population of bacteria at time 0. The number of bacteria then declines as $B(t) = 10,000e^{-2t}$, where t is measured in hours. In how many hours only 5% of the initial number of bacteria are left? (You may leave the answer in symbolic notation.)

$$B(0) = 10000 \cdot e^{-2 \cdot 0} = 10000. \quad (1)$$

3+

$$\frac{B(t)}{B(0)} = 0.05 \quad (1)$$

$$\frac{10000\, e^{-2t}}{10000} = 0.05$$

$$e^{-2t} = 0.05$$

$$-2t = \ln(0.05) \quad (1)$$

$$t = -\ln(0.05)/2 \sim \underline{1.5\ h}$$

5. For given constants a and b, consider the following function $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ ax+b & \text{if } 1 \le x \le 3 \\ |x-3| & \text{if } x > 3 \end{cases}$

a) **[3 marks]** Calculate $\lim_{x\to 1^-} f(x) =$ $\lim_{x\to 1^-} x^2 = \underline{\underline{1}}$; 1.5

1.5 +

make it integer

1.5

b) **[3 marks]** Calculate $\lim_{x\to 3^+} f(x) =$ $\lim_{x\to 3^+} |x-3| = \lim_{x\to 3^+} (x-3) = \underline{\underline{0}}$;

1.5 +

f must be continuous on $(-\infty,1)$, $[1,3]$, $(3,\infty)$

c) **[6 marks]** Determine constants a and b so that f is a continuous function.

1 · $f(x)$ defined and continuous on $(-\infty,1)$, $[1,3]$, $(3,\infty)$

· $\lim_{x\to 1^+} f(x) = a+b \overset{\text{we want}}{=\!=} 1 = \lim_{x\to 1^-} f(x) \Rightarrow a+b=1$ 1 1 1

· $\lim_{x\to 3^-} f(x) = 3a+b \overset{\text{we want}}{=\!=} 0 = \lim_{x\to 3^+} f(x) \Rightarrow 3a+b=0$ 1 1

$\begin{cases} a = -\frac{1}{2} \\ b = \frac{3}{2} \end{cases}$

$$\boxed{a = -\frac{1}{2}, \quad b = \frac{3}{2}}$$

13

tW