

Blue Solutions

Math 154, Calculus I for the Biological Sciences

Midterm I, Feb. 6

Instructor: Matt DeVos

First Name (print): _____
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1. DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
2. Fill in the above box.
3. Once the exam begins, check that your exam has 6 questions and 7 pages.
4. Only the usual writing instruments, this booklet, and a scientific calculators are allowed. **No graphing or programmable calculators are permitted.**
5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.
6. You may use the back of the previous page for rough work or if you run out of space.
7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Problem	Score	Value
1		10
2		8
3		12
4		8
5		6
6		6
Total:		50

Problem 1. (10 points) Mark each statement as true (**T**) or false (**F**)

F The functions $f(x) = \ln x$ and $g(x) = 10^x$ are inverse.

F The function $f(x) = \sin x$ is even.

T If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $-1, 1$ are in its range, then 0 is in its range.

T If f, g are continuous, then $f \circ g$ is continuous.

F The graph of an exponential function on a log-log plot forms a line.

Problem 2. (8 points) Let $f(x) = \ln(x - 10)$

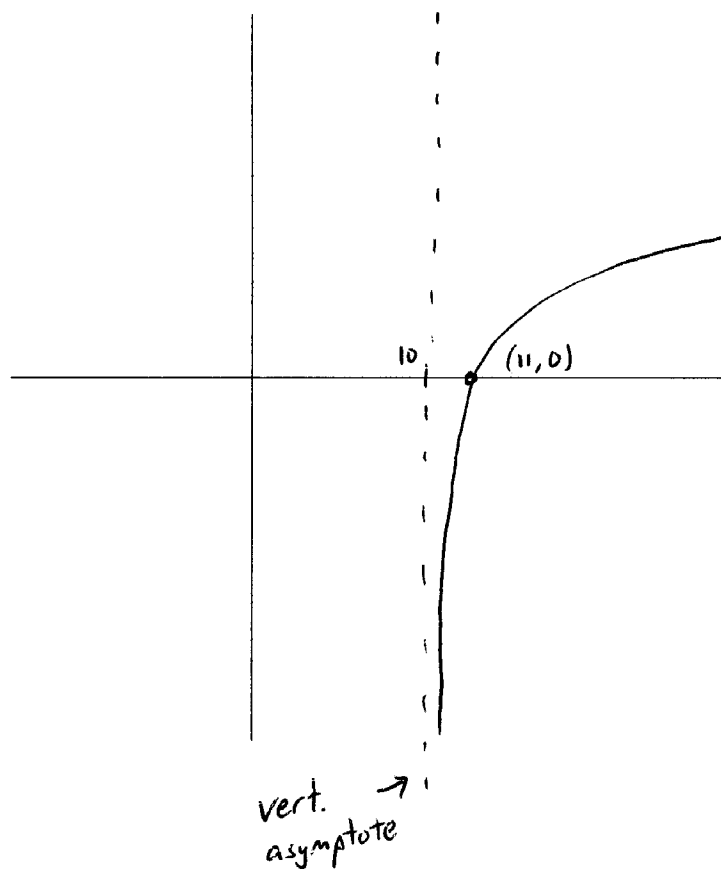
(2 points) What is the largest possible domain of f ?

$$\{x : x > 10\} = (10, \infty)$$

(2 points) Where is f continuous?

$$(10, \infty)$$

(4 points) Sketch the graph of f below, indicating all intercepts and asymptotes.



Problem 3. (12 points) Find the following limits.

(4 points)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 6x + 9}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)^5}{(x-3)(x-3)^2}$$

so the limit as $x \rightarrow 3^+$ is ∞
as $x \rightarrow 3^-$ is $-\infty$

D.N.E.

(4 points)

$$\lim_{x \rightarrow \infty} \frac{3x^5 + 2x^3 + x}{2x^5 + 4x^2 + 10}$$

$$= \lim_{x \rightarrow \infty} \frac{3x^5}{2x^5} = \frac{3}{2}$$

(4 points)

$$\lim_{x \rightarrow 0} e^x \frac{\sin 10x}{x}$$

$$= \lim_{x \rightarrow 0} e^x \frac{\sin 10x}{\left(\frac{1}{10}\right) 10x} = 10$$

Problem 4. (8 points) The decay of the element Neptunium (like any other radioactive particle) is governed by the equation $W(t) = W_0 e^{-\lambda t}$.

(5 points) If the half-life of Neptunium is 20 years, what is λ ? (you may leave this in symbolic form if you have no calculator).

W_0 is the initial amount

$$\frac{1}{2} W_0 = W(20) = W_0 e^{-\lambda \cdot 20}$$

$$\frac{1}{2} = e^{-20\lambda}$$

$$-\ln 2 = -20\lambda$$

$$\lambda = \frac{\ln 2}{20} \quad (\text{years}^{-1})$$

(3 points) If we begin with a sample of 50 grams, write an expression for the amount remaining after t years.

$$W_0 = 50$$

$$W(t) = W_0 e^{-\lambda t} = 50 e^{-\frac{\ln 2}{20} t}$$

(6 points) Consider the following function (here a, b are constants).

$$f(x) = \begin{cases} x^3 - x & \text{if } x < 2 \\ ax + b & \text{if } 2 \leq x \leq 4 \\ \frac{1}{4}x + 1 & \text{if } x > 4 \end{cases}$$

(1 point) Find $\lim_{x \rightarrow 2^-} f(x)$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^3 - x = 8 - 2 = 6$$

(1 point) Find $\lim_{x \rightarrow 4^+} f(x)$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} \frac{1}{4}x + 1 = 1 + 1 = 2$$

(4 points) Find values for a, b which make f a continuous function.

the line $ax + b$ must pass through $(2, 6)$ and $(4, 2)$

$$\text{so } a = \text{slope} = \frac{2 - 6}{4 - 2} = \frac{-4}{2} = -2$$

$$a = -2$$

$-2x + b$ passes thru $(2, 6)$

$$6 = -2 \cdot 2 + b$$

$$b = 10$$

6

$$\boxed{\begin{array}{l} a = -2 \\ b = 10 \end{array}}$$

Problem 6. (6 points) A strain of bacteria reproduces asexually every 10 minutes (so every 10 minutes, each bacterium splits in two). Assume that at the start there are 5 bacteria. Let a_0, a_1, a_2, \dots be the sequence given by the rule that a_i is the number of bacteria after $10i$ minutes.

(3 points) Find a recursive relationship for this sequence (i.e. express a_{i+1} as a function of a_i)

$$a_0 = 5$$

$$a_{i+1} = 2a_i$$

(3 points) Find a closed form equation for a_i (i.e. express a_i as a function of i).

i	0	1	2	3	...
a_i	5	$2 \cdot 5$	$2 \cdot 2 \cdot 5$	$2 \cdot 2 \cdot 2 \cdot 5$	

$$a_i = 5 \cdot 2^i$$