

# MATH 154 Calculus I for the Biological Sciences

## Midterm Exam 1

Professor: Robert Russell

Date: October 3, 2007

Time: 8:30 - 9:20AM

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

Signature: \_\_\_\_\_

- DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
- Write your name and student number on the front page.
- This exam is closed book and closed notes.
- **No graphing or programmable calculators are permitted.**
- You may lose marks if your explanations are incomplete or poorly presented.
- You may use the reverse side of the previous page for rough work or if you run out of space.
- During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.
- Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.
- This exam has 6 pages, including this cover page. Please make sure that you have all of the 6 pages.

Question	1	2	3	4	5	Total
Marks	/21	/24	/18	/16	/21	/100

**Problem 1** (21pt=8+7+6)

Consider the function defined by

$$f(x) = \ln(x^2 - 1) \quad \text{for } x > 1.$$

(i) Show the general shape of the graph of  $f(x)$ , showing any intercepts and asymptotes.

(ii) From the graph, how can you tell that the inverse  $f^{-1}(x)$  of  $f(x)$  exists? Find  $f^{-1}(x)$

(iii) Find the domain and the range of  $f^{-1}(x)$ .

**Problem 2** (24pt = 8+8+8)

Find the following limits, if they exist, using the limit laws discussed in class to justify your answers.

(i)

$$\lim_{x \rightarrow 0} e^{-x^2} \frac{\sin^2 x}{(2x)^2} =$$

(ii)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 36} - 6}{2x^2} =$$

(iii) For the sequence  $a_n = (\frac{n+e^{-n}}{n})$ ,  $n = 0, 1, 2, \dots$ ,

$$\lim_{n \rightarrow \infty} a_n =$$

**Problem 3** (18pt = 6+6+6)

Using the definition of a continuous function at a point, explain whether or not

$f(x) = \frac{x^2-4}{|x-2|}$  is continuous at the points

(i)  $x = -2$  and

(ii)  $x = +2$  .

(iii) When  $f(x)$  is discontinuous above, is it a removable discontinuity? Why?

**Problem 4** (16pt)

The average mass of the trees in a forest depends upon how dense the forest is. In particular, one finds that when considering the average mass of the trees as a function of the density of the surviving trees, the graph of the relationship using a log-log plot is a straight line with slope  $-\frac{5}{3}$ . If the mass is 1 kg when the density is  $10^4$  per square km, what is the equation for this straight line in the log-log plot? What is the relation between the average mass of the trees and their density?

**Problem 5** (21pt = 7+7+7)

(i) What is the Intermediate Value Theorem? If you cannot state it precisely, at least explain what it basically *says*.

(ii) Use the Intermediate Value Theorem to show that the polynomial

$$p(x) = 3x^3 - 4x^2 + 2$$

has at least one root and find an interval which contains that root.

(iii) How would you find an accurate approximation to the root? (You should not find a better approximation, just briefly explain the method you would use to find it.)