Simon Fraser University

MATH 154

Final Exam

April 16, 2005, 15:30 – 18:30

Last Name (please print):	______________________
First Name (please print):	______________________
Student Number:	______________________
Signature:	______________________

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 9 pages with a total of 9 questions. Once the exam begins please check to make sure your exam booklet is complete.
4. Only complete well-organized solution will receive full credit
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Question	Marks
1	/8
2	/6
3	/18
4	/12
5	/6
6	/12
7	/12
8	/8
9	/18
Total	**/100**

Good Luck

1 Answer **T** (true) or **F** (false) in the boxes provided. No explanation is necessary. Every correct answer will receive **1**. **[8 marks]**

a) ☐ $e^{x+y} = e^x e^y$, for all real numbers x and y.

b) ☐ If $f(-1) = -2$ and $f(1) = 2$ then there is at least one number c in the interval $(-1,1)$ such that $f(c) = 0$.

c) ☐ If $f'' > 0$ on the interval (a, b), then f is increasing on (a, b).

d) ☐ The domain of $y = \ln(\ln x)$ is all positive numbers.

e) ☐ $3x^2 + 2x$ is an antiderivative of $x^3 + x^2 + 1$.

f) ☐ If $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ exists, $\lim_{x\to a} f(x) = f(a)$.

g) ☐ If a continuous function f has a local extreme at $x = 1$, then $f'(1) = 0$.

h) ☐ $x = 0$ is a vertical asymptote to the graph of $f(x) = \frac{\sin x}{x}$.

2 Determine all numbers a such that $f(x) = \begin{cases} \frac{a|1-x|}{x-1}, x < 1 \\ -a^2, x \geq 1 \end{cases}$ is continuous.

[6 marks]

3 Find the limits: **[6 marks each]**

a) $\lim_{x\to 0}\frac{\sqrt[100]{1+x}-\sqrt[100]{1+2x}}{x}$

b) $\lim_{u\to -\infty}(\sqrt{u^2+2u}+u)$

c) $\lim_{x\to\infty}(1-\frac{2}{x})^{3x}$

4 Find $\dfrac{dy}{dx}$. **[6 marks each]**

a) $y = \ln(\sec x + \sqrt{x^2+1})$

b) $\dfrac{x}{y} = e^{-xy^2}$.

5 Solve the initial value problem

$$\frac{dN}{dt} = e^{-2t}, \text{ for } t \ge 0 \text{ with } N(0) = 18.$$ **[6 marks]**

6 **[6 marks each]**

a) Two people start biking from the same point. One bikes east at 15 m/h, the other north at 18 m/h. What is the rate at which the distance between the two people is changing after 30 minutes?

b) Find all tangent lines to the curve $y = -x^2$ that pass through the point $(0, 4)$.

7 Let $N(t)$ be the size of population at time t. **[6 marks each]**

a) Suppose that the per capita growth rate is equal to 1%, that is $\frac{1}{N}\frac{dN}{dt}=0.01$, and $N(5)=100$. Find a linear approximation of $N(t)$ at $t=5$ and use it to compute the approximate population size at time $t=5.1$.

b) Suppose $N(0)=50$ and $N'(t)\le 5$ for $t>0$. How large can $N(8)$ possibly be?

8 We need to design a cylindrical can. The top and bottom are made of copper, which will cost $3¢/\text{in}^2$. The curved side is to be made of aluminum, which will cost $2¢/\text{in}^2$. The total cost of the can is to be $120\pi¢$. Find the dimensions that will maximize the volume of the can. **[8 marks]**

9 Let $f(x) = e^{-x^2/2}$.

a) Identify the domain of f. **[2 marks]**

b) Locate the $y-$intercept and the $x-$intercepts of f. **[2 marks]**

c) Determine whether f is even or odd. **[2 marks]**

d) Determine the interval(s) on which f is increasing or decreasing and find the local extremes of f. **[4 marks]**

e) Determine the interval(s) on which f is concave up or do and find the inflection points of f. **[4 marks]**

f) Determine any asymptotes. **[2 marks]**

g) Sketch the graph of f. **[2 marks]**