Simon Fraser University

MATH 154 – FINAL EXAM

Instructor: N.Kouzniak

December 14, 2005

Last Name KEY

Given Name(s) SOLUTIONS

Student ID

Signature

INSTRUCTIONS

1. Do not open this booklet until instructed to do so. The booklet contains 14 pages including the cover page.

2. **Print** your name and student ID in the space provided above.

3. For each question you must **show all your work** unless stated otherwise.

4. No book, paper, or device other than the usual writing instruments, this booklet, and scientific calculators are allowed. **In particular, no graphing/programmable calculators are allowed.**

5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.

6. You may use the back of the previous page for a rough work or if you run out of space.

7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Question	Maximum	Mark
1	10	
2	4	
3	12	
4	16	
5	2	
6	4	
7	4	
8	4	
9	3	
10	5	
11	6	
12	4	
13	18	
14	4	
15	4	
Total	100	

1. [10 marks total] Complete the following theorems and definitions:

a) [2 marks] Suppose that $f(x)$ is continuous on $[a,b]$, differentiable on (a,b).

- If $f'(x)>0$ for all $x \in (a,b)$, then $f(x)$ is **increasing** on $[a,b]$, and
- If $f'(x)<0$ for all $x \in (a,b)$, then $f(x)$ is **decreasing** on $[a,b]$.

b) [3 marks] The **Mean Value Theorem** states that, if $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one number $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

c) [3 marks] The **Intermediate Value Theorem** states that, if $f(x)$ is continuous on $[a,b]$ and L is any real number with $f(a)<L<f(b)$ or $f(b)<L<f(a)$, then there exists at least one number $c \in (a,b)$ such that

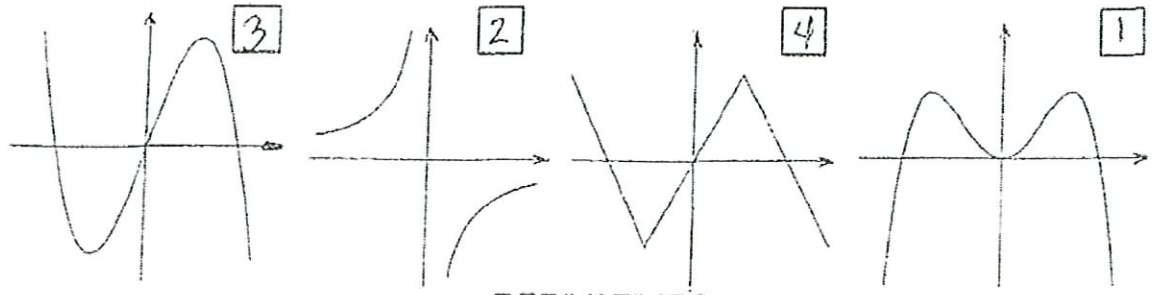
$$f(c)=L$$

d) [2 marks] **Fermat's Theorem** states that, if $f(x)$ has a local extremum at an interior point c and $f'(c)$ exists, then

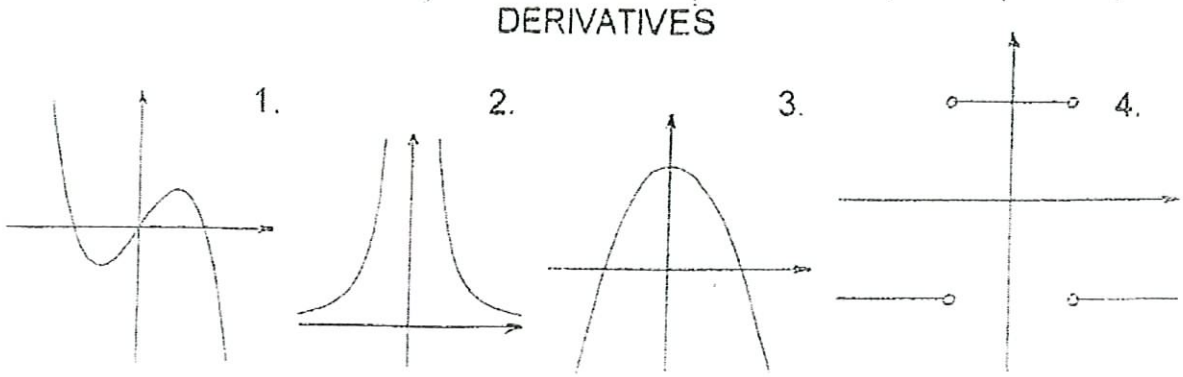
$$f'(c)=0$$

2. [4 marks] Below are the graphs of the functions and, in an arbitrary order, the graphs of their derivatives. Match each function with its derivative.

FUNCTIONS



DERIVATIVES



3. [12 marks total] Find the following limits:

a) $\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{X\to 2^-} \frac{X-2}{X-2} = 1$

b) $\lim_{x\to\infty} \frac{e^x}{x^2} \overset{\frac{\infty}{\infty}\,L'H}{=} \lim_{X\to\infty} \frac{e^X}{2X} \overset{\frac{\infty}{\infty}\,L'H}{=} \lim_{X\to\infty} \frac{e^X}{2} = \infty$

c) $\lim_{x\to\infty} \dfrac{x^2-2x+5}{-2x^2+x-8} = \lim_{x\to\infty} \dfrac{1-\frac{2}{x}+\frac{5}{x^2}}{-2+\frac{1}{x}-\frac{8}{x^2}} = -\dfrac{1}{2}$

d) $\lim_{x\to 0^+} (1+\sin 4x)^{\cot x}$

let $y = (1+\sin 4x)^{\cot x}$

$\ln y = \cot x \ln(1+\sin 4x)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \cot x \ln(1+\sin 4x) \overset{\infty\cdot 0}{=} \lim_{x\to 0^+} \dfrac{\ln(1+\sin 4x)}{\tan x}$

$\overset{\frac{0}{0}\ L'H}{=} \lim_{x\to 0^+} \dfrac{\frac{1}{1+\sin 4x}\cdot 4\cos 4x}{\sec^2 x} = \dfrac{\frac{1}{1+0}\cdot 4\cdot 1}{1^2} = 4$

So $\lim_{x\to 0^+} (1+\sin 4x)^{\cot x} = e^4$

e) $\lim_{x\to\infty}(xe^{1/x}-x) = \lim_{x\to\infty} x\left(e^{1/x}-1\right) \overset{\infty\cdot 0}{=} \lim_{x\to\infty} \dfrac{e^{1/x}-1}{\frac{1}{x}}$

$\overset{\frac{0}{0}\ L'H}{=} \lim_{x\to\infty} \dfrac{-x^{-2}e^{1/x}}{-x^{-2}} = \lim_{x\to\infty} e^{1/x} = 1$

f) $\lim_{x\to\infty}(x-\sqrt{x^2-1}) = \lim_{x\to\infty} 1-\sqrt{1-\frac{1}{x^2}} = 0$

OR

$$\lim_{x\to\infty} x-\sqrt{x^2-1} = \lim_{x\to\infty} x-\sqrt{x^2-1}\left(\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}} = 0$$

4. [16 marks total] Find $\frac{dy}{dx}$ for each of the following functions (DO NOT SIMPLIFY):

a) $y=\tan(\sqrt{x^2+\sqrt{3x-1}})$

$$dy/dx = \sec^2\left(\sqrt{x^2+\sqrt{3x-1}}\right)\cdot\frac{1}{2}\left(x^2+\sqrt{3x-1}\right)^{-1/2}\cdot\left(2x+3\cdot\frac{1}{2}(3x-1)^{-1/2}\right)$$

b) $y=\frac{\sin(x^2+2x)}{\cos^2 4x}$

$$dy/dx = \frac{\cos^2 4x\left(\cos(x^2+2x)\right)(2x+2) - \sin(x^2+2x)\left(-8\sin 4x\cdot\cos 4x\right)}{(\cos^2 4x)^2}$$

c) $y=3^{x-x^3}(x+e^{-4x})$

$$dy/dx = 3^{x-x^3}\left(1-4e^{-4x}\right) + 3^{x-x^3}\cdot\ln 3\cdot(1-3x^2)(x+e^{-4x})$$

7. [4 marks] Suppose that $f(x)$ is differentiable for all $x \in R$ and $f(x+y) = f(x)f(y)$. Also suppose that $\lim_{h\to 0} \frac{f(h)-1}{h} = 1$. Use this information and limit definition of the derivative to prove $f'(x) = f(x)$ and give an example of such a function.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(x)\cdot f(h) - f(x)}{h}$$

$$= \lim_{h\to 0} f(x)\left(\frac{f(h)-1}{h}\right) = f(x)\cdot \lim_{h\to 0} \frac{f(h)-1}{f(h)} = f(x)\cdot 1 = f(x).$$

$f(x) = e^x$ is an example of a function with $f(x) = f'(x)$.

8. [4 marks] Find the equation of a line tangent to the graph of the curve defined implicitly $x^2 + 2xy^2 + 3y^4 = 6$ at the point (1, -1).

$$2x + 2y^2 + 4xyy' + 12y^3y' = 0$$

$$y' = \frac{-2x-2y^2}{4xy+12y^3}$$

$$y'|_{(1,-1)} = \frac{-4}{-16} = \frac{1}{4}$$

$$y+1 = \frac{1}{4}(x-1)$$

The tangent line equation is $y = \frac{1}{4}x - \frac{5}{4}$