Simon Fraser University
Department of Mathematics
Math 154-3, Calculus I for Biological Sciences
Final Examination

Instructor: E. Lee **16 Dec 2002**

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Last Name Student Number

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Given Names

Instructions

1 Do not turn over this page until instructed to do so.

2 No calculators allowed.

3 Full marks will be awarded for correct, complete and well-organized solutions.

4 Except for Question 1, full justification of answers is required, and requested methods must be used.

5 Check that there are 19 pages in this examination booklet.

6 This examination consists of nine questions. You have three hours to complete this examination.

For examiner's use only

Question	Marks
1	**/14**
2	**/7**
3	**/5**
4	**/5**
5	**/4**

Question	Marks
6	**/8**
7	**/5**
8	**/4**
9	**/13**
Total	**/65**

1. Multiple choice questions. Circle clearly your answers to the multiple questions below. [14 marks]

(A)	a	b	c	d	e
(B)	a	b	c	d	e
(C)	a	b	c	d	e
(D)	a	b	c	d	e
(E)	a	b	c	d	e
(F)	a	b	c	d	e
(G)	a	b	c	d	e
(H)	a	b	c	d	e
(I)	a	b	c	d	e
(J)	a	b	c	d	e
(K)	a	b	c	d	e
(L)	a	b	c	d	e
(M)	a	b	c	d	e
(N)	a	b	c	d	e

(A) The circle described by the equation

$$x^2 + y^2 - 2x + 4y = 4$$

has center (a, b) and radius r. The respective values of a, b and r are

(a) -2, 4 and 2

(b) 1, -2 and 3

(c) 2, -4 and 4

(d) -1, 2 and 3

(e) -2, 4 and $\sqrt{2}$

(B) Evaluate

$$\lim_{x \to \infty} \frac{\exp\left(\frac{1}{x}\right) - \exp(-x) + \exp\left(-\frac{1}{x^2}\right)}{\exp(-\sqrt{x}) + \exp\left(\frac{1}{\sqrt{x}}\right)}$$

(a) $-\infty$

(b) 0

(c) 1

(d) 2

(e) ∞

(C) The derivative of

$$y = \ln\left(\ln\left(2^x\right)\right)$$

with respect to x is

(a) $1/x$

(b) $(\ln 2)/x$

(c) $1/\ln\left(2^x\right)$

(d) $2^x/\ln\left(2^x\right)$

(e) $\ln\left(1/\left(2^x\right)\right)$

(D) Evaluate

$$\frac{d}{dx}\sin\left(x^2\right)$$

(a) $2x\cos(x)$

(b) $2x\sin\left(x^2\right)$

(c) $2x\cos\left(x^2\right)$

(d) $2x\sin(2x)$

(e) $2x\cos(2x)$

(E) Evaluate

$$\frac{\ln 1 - \ln 3 + \ln 5 + \ln 135}{\ln 1 + \ln 3 + \ln 5 + \ln 15}$$

(a) 1

(b) 3

(c) 5

(d) 1/3

(e) none of the above

(F) Evaluate

$$\lim_{x \to 0} (\csc x - \cot x \cos x)$$

(a) 1

(b) -1

(c) ∞

(d) 0

(e) none of the above

(G) Determine where the function

$$f(x) = \begin{cases} |1+x| + |1-x| & \text{if } x > 0 \\ 2 - x^2 & \text{if } x \leq 0 \end{cases}$$

is not differentiable.

(a) $x = -1$

(b) $x = 1$

(c) $x = 0$

(d) $x = -1, 1$

(e) $x = 0, 1$

(H) Let $f(x)$ be a function that is continuous everywhere. Suppose a, b, c, d are numbers such that

$$a < b < c < d \quad \text{and} \quad f(a) < f(c) < 0 < f(b) < f(d)$$

Then the equation $f(x) = 0$ has at least n solutions. What is n ?

(a) 2

(b) 3

(c) 4

(d) 5

(e) none of the above

(I) What is the domain of the following function?

$$f(x) = \sqrt{\frac{x^3}{3} - x}$$

(a) $\left[-\sqrt{3}, \sqrt{3}\ \right]$

(b) $\left(-\infty, -\sqrt{3}\ \right]$ and $\left[0, \sqrt{3}\ \right]$

(c) $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$

(d) $\left(-\infty, -\sqrt{3}\ \right]$ and $\left[\sqrt{3}, \infty\right)$

(e) $\left[-\sqrt{3}, 0\right]$ and $\left[\sqrt{3}, \infty\right)$

(J) Evaluate

$$\lim_{x \to 0-} \frac{\ln(e + x) - 1}{x}$$

(a) 0

(b) ∞

(c) e

(d) $-e$

(e) $1/e$

(K) Evaluate

$$\lim_{x \to 0+} \frac{\ln x}{x}$$

(a) ∞

(b) 0

(c) 1

(d) -1

(e) none of the above

(L) Let m_1 be the slope of the tangent line of

$$f(x) = \pi x^2 + e^x \sin x$$

at $x = 0$, and let m_2 be the slope of the normal line of $f(x)$ at $x = 0$. Evaluate $m_1 + m_2$.

(a) -1

(b) 1

(c) 0

(d) e

(e) π

(M) Let $f(x)$ be a function that is differentiable everywhere. Assume

$$f(1) = \pi^2, \quad f(\pi) = 1.57, \quad f'(1) = -\pi, \quad f'(\pi) = 3.14$$

Evaluate

$$\frac{d}{dx} f\left(\sqrt{f(x^2)}\right)$$

at $x = -1$.

(a) $-1/2$

(b) π

(c) 1.57

(d) 3.14

(e) none of the above

(N) On a certain day, a scientist had 1 kg of a radioactive substance X at 12:00 pm. After 6 hours, only 64 g of substance X remained. How much substance X was there at 2 pm the same day?

(a) 200 g

(b) 400 g

(c) $1000 \exp\left(\frac{1}{3} \ln \frac{125}{8}\right)$ g

(d) $1000 \exp\left(\frac{1}{6} \ln \frac{8}{125}\right)$ g

(e) none of the above

2. Evaluate the following limits.

(A) $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$ [3 marks]

(B) $\lim_{x\to 0-}\left(\frac{1}{x}-\frac{1}{\tan x}\right)$ [4 marks]

3. Suppose

$$e^y = \cos(x + \sin y)$$

Find $\dfrac{dy}{dx}$ by implicit differentiation. [5 marks]

4. Suppose

$$y = \frac{e^x \sqrt{1+x}}{(3x-2)^3 \sin^2 x}$$

Find $\dfrac{dy}{dx}$ by logarithmic differentiation. [5 marks]

5. Determine where the function

$$f(x) = \begin{cases} x \sin(1/x) & \text{if } x > 0 \\ \cos^2 x & \text{if } x \leq 0 \end{cases}$$

is continuous. Justify your answer. [4 marks]

6. A clock has an hour hand that is 5 cm long and a minute hand that is 8 cm long. How fast is the distance between the tip of these two hands changing at 2:00 am? [8 marks]

7. The Ricker curve

$$R(x) = \alpha x e^{-\beta x}, \qquad x \geq 0$$

where α and β are positive constants, describes the relationship between the size x of the parental stock of some fish and the number R of recruits. Suppose $\alpha = 1/3$ and $\beta = 1/2$. Find the size of the parental stock that will maximize recruit. What is the maximum number of recruits? [5 marks]

8. Let $f(x)$ be a function differentiable on $(0, \infty)$. Suppose $f(1/2) = 1$ and

$$f'(x) = \sin(\pi x) - \frac{\pi}{x}$$

Find $f(x)$. [4 marks]

9. Let

$$f(x) = (5 - 2x)\, x^{2/3}, \qquad x \text{ in } (-\infty, \infty)$$

(A) Does $f(x)$ have any asymptotes? Justify your answer. [2 marks]

(B) Given that

$$f'(x) = \frac{10}{3}\left(\frac{1-x}{x^{1/3}}\right)$$

Determine where $f(x)$ is increasing and where it is decreasing. Locate all critical points and local extrema of $f(x)$. [4 marks]

(C) Given that

$$f''(x) = -\frac{10}{9}\left(\frac{1+2x}{x^{4/3}}\right)$$

Determine where $f(x)$ is concave up and where it is concave down. Locate all inflection points of $f(x)$. [3 marks]

(D) Use the information in **(A)**, **(B)** and **(C)** to sketch a graph of $f(x)$. [4 marks]

Special limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0 \qquad \lim_{x\to 0}\frac{a^x-1}{x}=\ln a$$

Exact trigonometric values

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1

Trigonometric identities

$$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$$

$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$

$$\sin^2 x+\cos^2 x=1$$

$$\sin(2x)=2\sin x\cos x$$

$$\cos(2x)=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$$

Derivative formula

$$\frac{d}{dx}\sin x=\cos x \qquad \frac{d}{dx}\csc x=-\csc x\cot x$$

$$\frac{d}{dx}\cos x=-\sin x \qquad \frac{d}{dx}\sec x=\sec x\tan x$$

$$\frac{d}{dx}\tan x=\sec^2 x \qquad \frac{d}{dx}\cot x=-\csc^2 x$$

$$\frac{d}{dx}a^x=(\ln a)\,a^x \qquad \frac{d}{dx}\log_a x=\frac{1}{(\ln a)\,x}$$

Let a,b,c be the three sides of a triangle, and let A,B,C be the angles opposite a,b,c respectively.

Law of Cosine
$$\begin{cases} a^2=b^2+c^2-2bc\cos A\\ b^2=a^2+c^2-2ac\cos B\\ c^2=a^2+b^2-2ab\cos C\end{cases}$$

Law of Sine
$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$