Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, Calculus II
Summer 2006 – Midterm 2
July 5^{th}, 2006, 8:30 – 9:20

Last Name (please print): KEY

First Name (please print): ______________________

SFU email ID: ______________________

Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 3 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **No** calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1 a),b)	/6
1 c),d)	/6
1 e),f)	/6
2	/6
3	/16
Total	**/40**

1. Compute the following indefinite integrals and describe which methods you used. **[3 marks each = 18 marks]**

a) $\int \frac{1}{x^2 - 5x + 6} dx$ $= \int \frac{1}{(x-3)(x-2)} dx$

$= \int \frac{1}{x-3} - \frac{1}{x-2} dx$

$= \ln|x-3| - \ln|x-2| + C$

b) $\int \frac{x^3 - 2\sqrt[3]{x} + 5}{\sqrt{x}} dx$

$= \int x^{2.5} - 2x^{-\frac{1}{6}} + 5x^{-\frac{1}{2}} dx$

$= \frac{x^{3.5}}{3.5} - \frac{12}{5} x^{\frac{5}{6}} + 10 x^{\frac{1}{2}} + C$

c) $\int \sin^3 x \cos^3 x \, dx$

$$= \int \sin^3 x \, (1 - \sin^2 x) \cos x \; dx$$

$$= \int (\sin^3 x - \sin^5 x) \cos x \; dx$$

$$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$$

$$\left(\text{OR} \quad \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C \right)$$

d) $\int e^{2x} \sqrt{e^{2x}+1} \, dx$

$$w = e^{2x} + 1$$

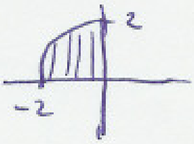
$$dw = 2e^{2x} \, dx$$

$$= \frac{1}{2} \int \sqrt{w} \; dw$$

$$= \frac{1}{2} \cdot \frac{2}{3} w^{\frac{3}{2}} + C$$

$$= \frac{1}{3} (e^{2x} + 1)^{3/2} + C$$

e) $\int_{-2}^{0} \sqrt{4-x^2}\, dx$

$y = \sqrt{4-x^2}$, $-2 \le x \le 0$ is the quarter circle  with area π.

So, $\int_{-2}^{0} \sqrt{4-x^2}\, dx = \pi$.

f) $\int_0^1 x^2 e^x\, dx$

$u = x^2 \qquad dv = e^x\, dx$

$du = 2x\, dx \qquad v = e^x$

$= x^2 e^x - 2\int_0^1 x e^x\, dx \qquad u = x \qquad ''$

$du = dx$

$= x^2 e^x - 2x e^x + 2\int_0^1 e^x\, dx$

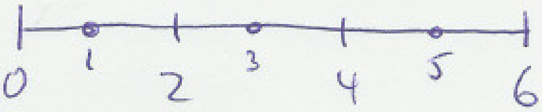
$= x^2 e^x - 2x e^x + 2e^x \Big]_0^1$

$= (e - 2e + 2e) - (0 - 0 + 2)$

$= e - 2$

2. Use Midpoint Rule with $n=3$.

a) Approximate $\int_0^6 x^2\,dx$. **[4 marks]**



$$\Delta = \frac{6-0}{3} = 2$$

$$\int_0^6 x^2\,dx \approx M_3 = 2\left(1^2 + 3^2 + 5^2\right) = 70$$

b) How large do you have to choose n so that the approximation in part a) is accurate to within 0.1? **[2 marks]**

$f'(x) = 2x$, $f''(x) = 2$, $|f''(x)| \leq 2 = K$

$$\frac{2(6-0)^3}{24n^2} \leq \frac{1}{10}$$

$180 \leq n^2$ Choose $n=14$.

3. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. **[2 marks each = 16 marks]**

a) $\boxed{\text{F}}$ $x = 3\sin\theta$ is the best trigonometric substitution to choose for the evaluation of $\int \frac{1}{\sqrt{x^2-9}}\,dx$.

b) $\boxed{\text{T}}$ The error bound for the Midpoint Rule is $|E_M| \le \frac{K(b-a)^3}{24n^2}$, where $|f''(x)| \le K$ for $a \le x \le b$.

c) $\boxed{\text{F}}$ The Trapezoidal Rule is

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right],$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

d) $\boxed{\text{T}}$ The improper integral $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$ is convergent.

e) $\boxed{\text{F}}$ The improper integral $\int_e^\infty \frac{1}{x\ln x}\,dx$ is convergent.

f) $\boxed{\text{F}}$ $\int_0^{\pi/4} \sqrt{1+\sec^2 x}\,dx$ computes the arc length of $y = \tan x$ for $0 \le x \le \pi/4$.

g) $\boxed{\text{T}}$ Let $y = f(x)$ be positive and smooth, then $\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$ computes the surface area of the solid obtained by rotating $y = f(x), a \le x \le b$ about the *x*-axis.

h) $\boxed{\text{T}}$ Let $y = f(x)$ be positive and smooth, then ds is the arc length differential described by $\sqrt{1+[f'(x)]^2}\,dx$.