Simon Fraser University
Department of Mathematics
Burnaby and Surrey Campus

MATH 152-3, Fall 2005
Midterm 2
November 2nd, 2005, 8:30 – 9:20

Last Name (please print): KEY

First Name (please print): ____________

Student Number: ____________

Instructor: B. Kadonoff I. Mercer

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

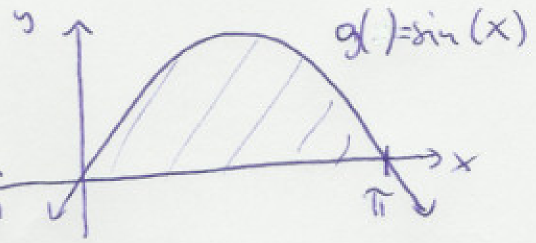
Do not write in this table!	
Question	**Marks**
1	/3
2	/4
3 a, b	/10
3 c, d	/10
4	/3
Total	**/30**

1. The solid of revolution A is formed by rotating $f(x)=\cos(x)$, $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$ about the x-axis. The solid of revolution B is formed by rotating $g(x)=\sin(x)$, $0\le x\le\pi$ about the x-axis. **[3 marks]**

a) Why do these two solids have equal surface area?

Because the graph of g is just the graph of f shifted to the right by $\frac{\pi}{2}$.

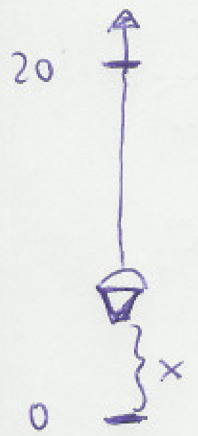
b) Set up **but do not** evaluate, an integral to compute the surface area of **one** of these solids.



$$S=\int_a^b 2\pi\, g(x)\sqrt{1+[g'(x)]^2}\,dx$$

$$=\int_0^{\pi} 2\pi \sin(x)\sqrt{1+\cos^2(x)}\,dx$$

2. A bucket that weighs 35 kg when filled with water is lifted at a constant rate from the bottom of a well that is 20 meters deep. The chain that is being used to lift the bucket weighs 0.5 kg per meter. Compute the work required to lift the bucket from the bottom of the well to the top. Recall, $g = 9.8\,\text{m/s}^2$. **[4 marks]**



$$F = mg \qquad W = \int_a^b F(x)\,dx$$

$$m = 35 + 0.5(20 - x) = 45 - 0.5x$$

$$W = \int_0^{20} (45 - 0.5x)\,9.8\,dx$$

$$= 9.8\left[45x - 0.25x^2\right]_0^{20}$$

$$= 9.8\,[900 - 100]$$

$$= 7840 \text{ J}$$

OR $\quad W = \int_0^{20} 9.8(35 - 0.5x)\,dx$

3. Evaluate the following indefinite integrals. **[5 marks each = 20 marks]**

a) $\int x^2 \cos(x)\,dx$

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x\,dx \qquad v = \sin x$

$= x^2 \sin x - 2\int x \sin x \, dx$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$= x^2 \sin x + 2x\cos x - 2\int \cos x \, dx$

$= x^2 \sin x + 2x \cos x - 2 \sin x + C$

b) $\int \frac{dx}{x^3+x} = I$

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

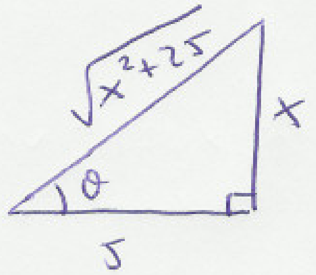
$$I = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + C$$

c) $\int \frac{1}{\left(x^2+25\right)^{3/2}}\,dx$

$x = 5\tan\theta$

$dx = 5\sec^2\theta\, d\theta$



$$= \int \frac{5\sec^2\theta\, d\theta}{(25\sec^2\theta)^{3/2}}$$

$$= \frac{1}{25}\int \frac{1}{\sec\theta}\, d\theta = \frac{1}{25}\int \cos\theta\, d\theta$$

$$= \frac{1}{25}\sin\theta + C = \frac{x}{25\sqrt{x^2+25}} + C$$

d) $\int \frac{1}{4x^2+4x-3}\,dx$

$$= \int \frac{1}{(2x-1)(2x+3)}\, dx$$

$$\frac{1}{(2x-1)}\cdot\frac{1}{(2x+3)} = \frac{\frac{1}{4}}{2x-1} - \frac{\frac{1}{4}}{2x+3}$$

$$= \frac{1}{4}\int \frac{1}{2x-1} - \frac{1}{2x+3}\, dx$$

$$= \frac{1}{8}\left[\ln|2x-1| - \ln|2x+3|\right] + C$$

4. Evaluate the definite integral $\int_{-1}^{8} \frac{1}{x^{2/3}} dx$, if it exists. **[3 marks]**

$$\int_{-1}^{8} \frac{1}{x^{2/3}} dx = \int_{-1}^{0} \frac{1}{x^{2/3}} dx + \int_{0}^{8} \frac{1}{x^{2/3}} dx$$

$$\int_{-1}^{0} \frac{1}{x^{2/3}} dx = \lim_{t \to 0^-} \int_{-1}^{t} x^{-2/3} dx$$

$$= \lim_{t \to 0^-} \left[3x^{1/3} \right]_{-1}^{t}$$

$$= 3 \lim_{t \to 0^-} \left[t^{1/3} + 1 \right]$$

$$= 3$$

Similarly,

$$\int_{0}^{8} \frac{1}{x^{2/3}} dx = \lim_{t \to 0^+} \left[3x^{1/3} \right]_{t}^{8}$$

$$= 3 \lim_{t \to 0^+} \left[2 - t^{1/3} \right]$$

$$= 6$$

So, $\int_{-1}^{8} \frac{1}{x^{2/3}} dx = 3 + 6 = 9$