Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, Calculus II
Summer 2006 – Midterm 1
June 7^{th}, 2006, 8:30 – 9:20

Last Name (please print):	KEY
First Name (please print):	
SFU email ID:	
Instructor:	P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1	/14
2	/4
3	/2
4	/7
5	/7
6	/6
Total	**/40**

1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. **[2 marks each = 14 marks]**

a) $\boxed{\text{T}}$ For any function f defined on $[a,b]$ the definite integral is $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \Delta x\, f(x_i^*)$, if the limit exists and where $[a,b]$ is divided into n subintervals, $\Delta x = \dfrac{b-a}{n}$ and $x_i^* \in [x_{i-1}, x_i]$.

b) $\boxed{\text{T}}$ If f is a continuous function then $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$.

c) $\boxed{\text{F}}$ If $v(t)$ is the velocity of some object, then $\int_a^b v(t)\,dt$ is the total distance the object traveled for $a \le t \le b$.

d) $\boxed{\text{F}}$ $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$.

e) $\boxed{\text{T}}$ If $r = f(\theta)$ is a positive continuous function, then the area within the curve f for $\alpha \le \theta \le \beta$ is $A = \int_\alpha^\beta \frac{1}{2}[f(\theta)]^2\,d\theta$.

f) $\boxed{\text{T}}$ Suppose $y = f(x)$ bounded by $x = a$ and $x = b$ with $0 \le a \le b$ is rotated about the y-axis. Then the volume is found by $V = \int_a^b 2\pi x\, f(x)\,dx$.

g) $\boxed{\text{F}}$ If the function f is continuous on $[a,b]$, then there exists a number c in $[a,b]$ such that $f(c) = f_{avg} = \dfrac{1}{a-b}\int_a^b f(x)\,dx$.

2. The speed of a runner increased steadily during the first 3 seconds of a race. Her speed at half-second intervals is given in the table below. Approximate the distance that she traveled using three equal subintervals with left endpoints. **[4 marks]**

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v(ft/s)	0	4.9	9.2	13.7	15.5	16.1	16.6

$\Delta x = 1$

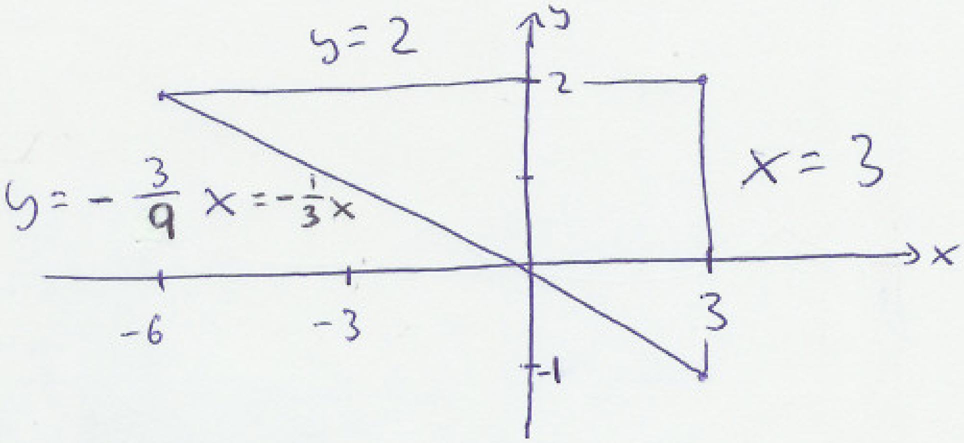
distance $\approx 1 \cdot (0 + 9.2 + 15.5) = 24.7$ ft

3. Given $f(x) = \int_4^{\sqrt{x}} \tan^2(t)\,dt$, find $\dfrac{df}{dx}$. **[2 marks]**

$$\frac{df}{dx} = \tan^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

4. Suppose a triangle is given with the vertices $(-6,2),(3,2),(3,-1)$.

a) Graph the triangle and label all three sides with line equations. **[3 marks]**



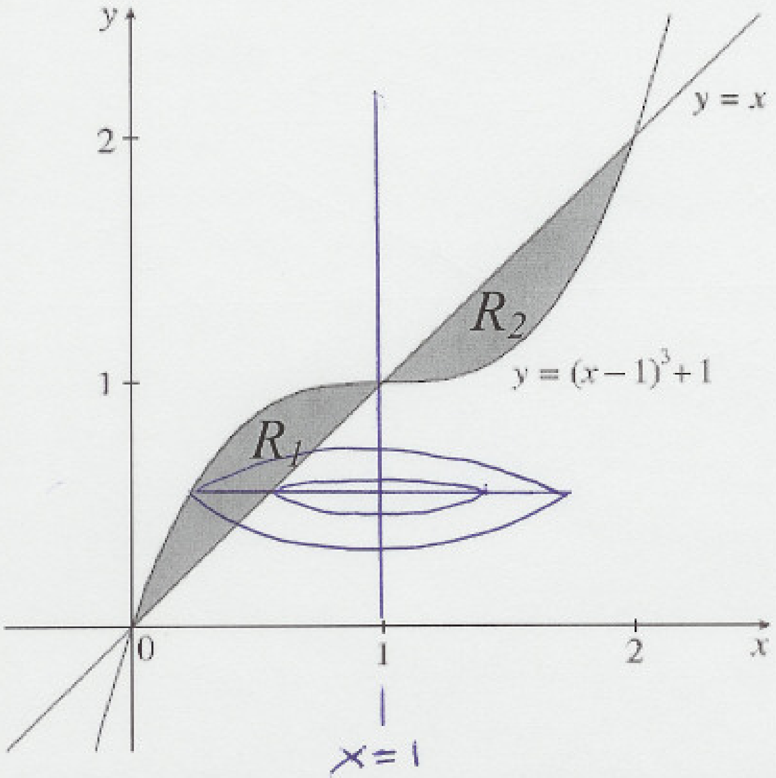
b) Set up but do **NOT** compute an integral for the area of the triangle. Do **NOT** simplify. **[4 marks]**

$$A(\text{triangle}) = \int_{-6}^{3} 2-\left(-\frac{1}{3}x\right)\,dx$$

OR

$$A(\text{triangle}) = \int_{-1}^{2} 3-(-3y)\,dy$$

5. Consider the region R_1 below. A solid S_1 is created by rotating the region about the line $x = 1$. Use washer method to answer the questions below.



$\Rightarrow x = y$

$\Rightarrow x = \sqrt[3]{y-1} + 1$

a) Draw the axis of rotation and one general cross-section into the above coordinate system. **[2 marks]**

b) Set up but do **NOT** compute an integral for the volume V_1 of the solid S_1. Do **NOT** simplify. **[4 marks]**

$$V_1 = \int_0^1 \pi \left[\left(1 - \left(\sqrt[3]{y-1} + 1\right)\right)^2 - (1-y)^2 \right] dy$$

c) Suppose R_2 is also rotated about the line $x = 1$ to create a solid S_2. Is the volume V_2 of the solid S_2 greater than, equal to, or less than V_1? **[1 mark]**

6. A force of 6 *N* is required to hold a spring stretched 5 meters beyond its natural length. How much work is done in stretching it from its natural length to 8 meters beyond its natural length? **[6 marks]**

$$F(x) = kx$$

$$6 = k5 \Rightarrow k = \frac{6}{5}$$

$$F(x) = \frac{6}{5}x$$

$$W = \int_0^8 \frac{6}{5}x \, dx$$

$$= \frac{3}{5}x^2 \Big]_0^8$$

$$= \frac{3 \cdot 64}{5}$$

$$= \frac{192}{5} \text{ J}$$

$$(= 38{,}4 \text{ J})$$