Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, Calculus II
Spring 2006 – Midterm 1
October 5th, 2005, 8:30 – 9:20

Last Name (please print): KEY

First Name (please print): _____

SFU email ID: _____

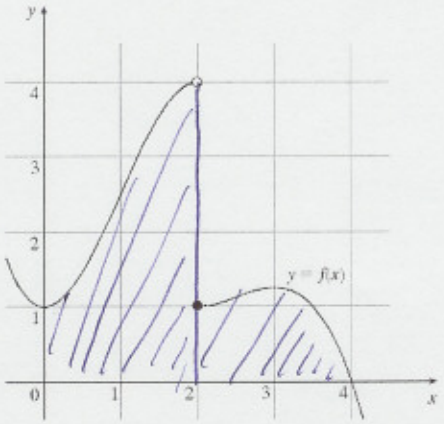
Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1	/8
2	/4
3	/7
4	/6
5	/5
Total	/30

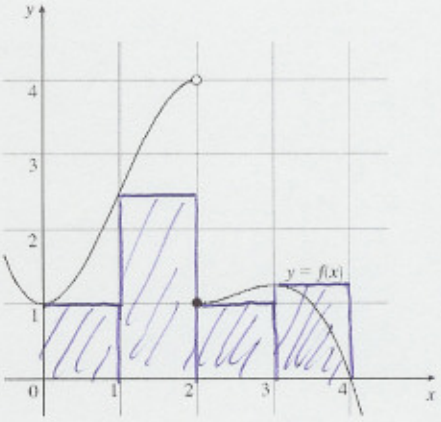
1. Let $f(x)$ be the piecewise continuous function graphed below and $I = \int_0^4 f(x)\,dx$.



≈ 7 square units

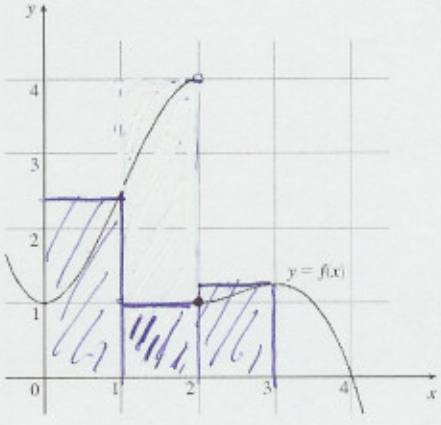
a) Show on the graph above what I represents graphically. **[1 mark]**

b) Compute L_4, the left-endpoint Riemann sum approximation of I, and show on the graph to the right what L_4 represents graphically. **[3 mark]**



$L_4 \approx 5.75$ square units

c) Compute R_4, the left-endpoint Riemann sum approximation of I, and show on the graph to the right what R_4 represents graphically. **[3 mark]**



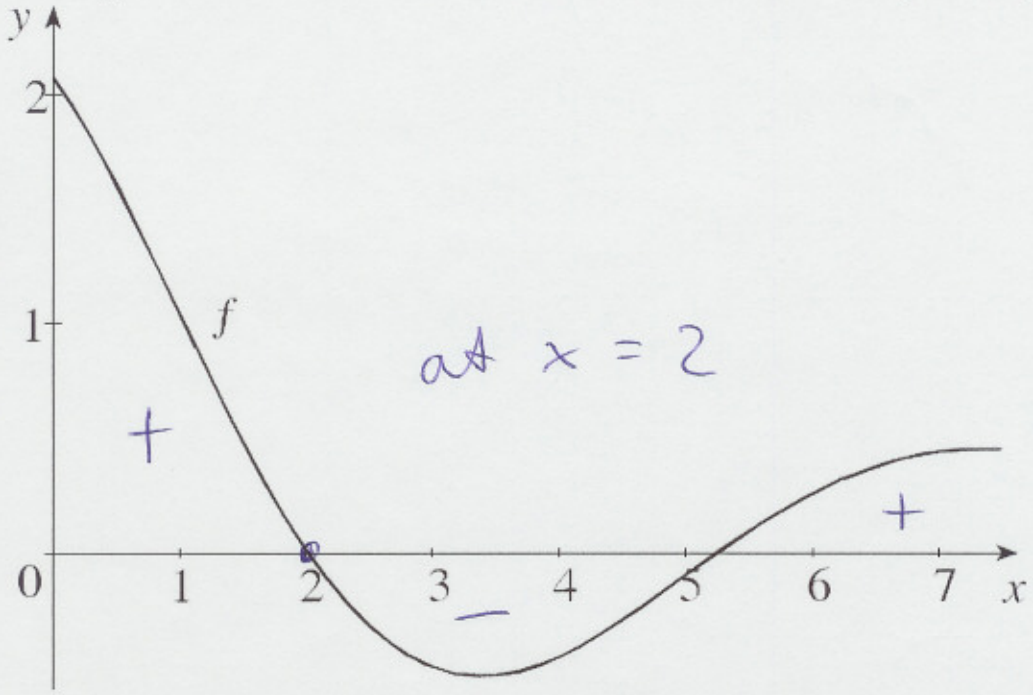
$R_4 \approx 4.50$ square units

d) Which of the approximations L_4 and R_4 is closer to the actual value of I? **[1 mark]**

L_4

2.

a) Let $g(x)=\int_0^x f(t)\,dt$, $0\le x\le 6$ where f is the graph below. At what value of x does g achieve its absolute maximum value on $0\le x\le 6$? **[2 mark]**



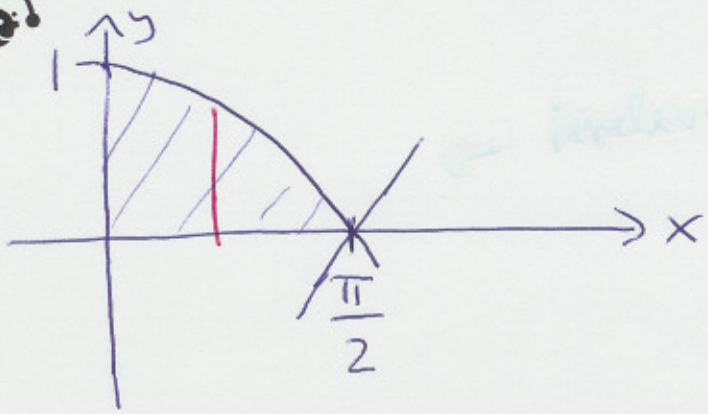
at $x = 2$

b) Let $g(x)=\int_3^{x^2} \ln(7+\sqrt{t})\,dt$. Find $g'(x)$. **[2 marks]**

$g'(x) = \ln(7+\sqrt{x^2})\cdot 2x$

3. Consider the curves $y = \cos x$, $y = \frac{4}{\pi}x - 2$ and $x = 0$ in the first quadrant.

a) Sketch the curves and shade the area they bound. **[2 marks]**



b) Set up an integral to find the area. **[2 marks]**

$$I = \int_0^{\frac{\pi}{2}} \cos x \; dx$$

c) Compute the integral. **[3 marks]**

$$I = [\sin x]_0^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin 0$$

$$= 1 - 0$$

$$= 1 \text{ s.u.}$$

4. Consider the region below. Set up but do **not** compute an integral for the volume obtained by rotating the region below about the line $y = -1$. Do not simplify the integral. **[6 marks]**

$$V = \pi \int_0^1 \left(\left((x+1)^3 + 1 + 1 \right)^2 - (x+1)^2 \right) dx$$

$$+ \pi \int_1^2 \left((x+1)^2 - \left((x+1)^3 + 1 + 1 \right)^2 \right) dx$$

5. A 5 pound bucket is lifted from the ground into the air by pulling in 20 feet of rope at a constant speed. The rope weighs 0.08 lb/ft. How much work was spent lifting the bucket and rope? **[5 marks]**

rope: $(20-y)\cdot 0.08$ bucket: 5

$$W = \int_0^{20} (20-y)\cdot 0.08 + 5 \; dy$$

$$= \int_0^{20} 1.6 - 0.08y + 5 \; dy$$

$$= 1.6y - 0.04y^2 + 5y \Big]_0^{20}$$

$$= 32 - 16 + 100$$

$$= 116 \text{ ft-lb}$$