Simon Fraser University
Department of Mathematics
Burnaby and Surrey Campus

MATH 152-3, Fall 2005
Midterm 1
October 5th, 2005, 8:30 – 9:20

Last Name (please print): KEY

First Name (please print): ______

Student Number: ______

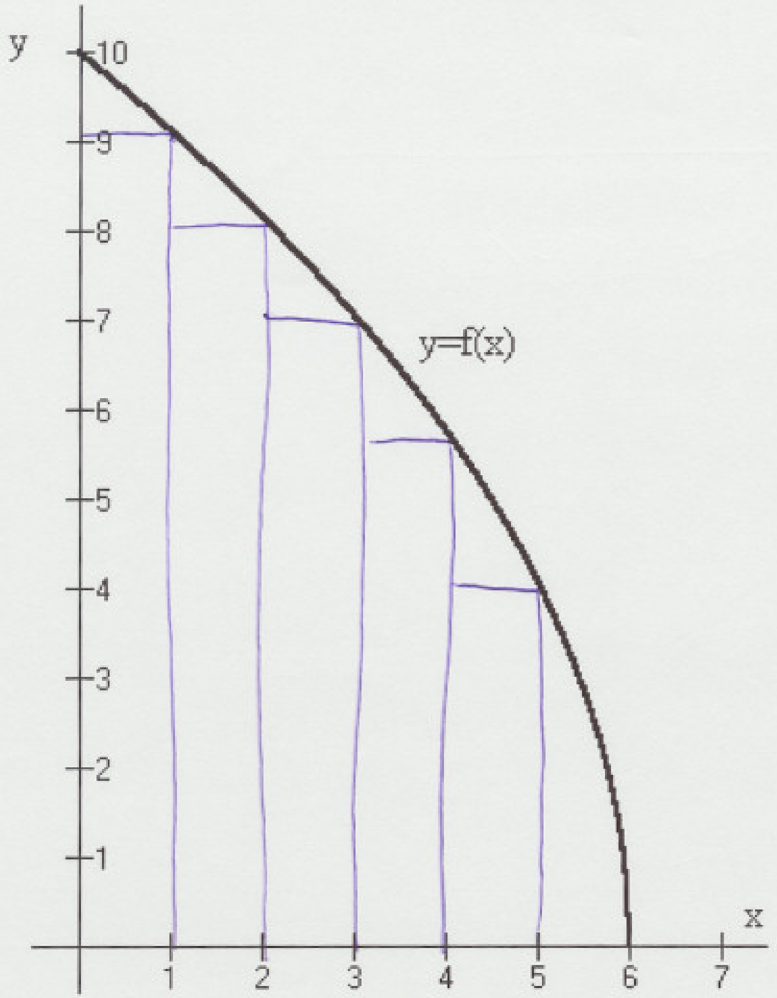
Instructor: B. Kadonoff I. Mercer

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1	/6
2	/4
3	/4
4	/6
5	/5
6	/5
Total	**/30**

1. Given the graph of f below, estimate the area under the graph of f on $[0,6]$ using Riemann sum with a regular partition of the given interval into 6 subintervals and right endpoints. In your estimates, use integers to the nearest unit. Illustrate your method on the graph. **[4 marks]**



$\Delta x = 1$

$R_6 = 1 \cdot (9 + 8 + 7 + 6 + 4) = 34$

2.

a) Given $f(x)=\int_{-4}^{x^3}\left[e^u+\cos(2u)\right]du$, find $f'(x)=\dfrac{df(x)}{dx}$. **[4 marks]**

$$f'(x) = 3x^2\left(e^{x^3} + \cos 2x^3\right)$$

b) If h' is a cedar tree's rate of growth in centimeters per year, circle all of the following expressions which represent the increase in the tree's height in centimeters between the years 10 and 20? No explanation needed. **[2 marks]**

i) $20-10$

ii) $\left.\dfrac{dh'}{dt}\right|_{t=20} - \left.\dfrac{dh'}{dt}\right|_{t=10}$

iii) $h(20)-h(10)$

iv) $h'(20)-h'(10)$

v) $\int_{10}^{20} h(t)\,dt$

vi) $\int_{10}^{20} h'(t)\,dt$

3. Compute the indefinite integrals. Show your substitution. **[4 marks]**

a) $\int \tan^3 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int \tan^3 x \sec^2 x \, dx$

$= \int u^3 \, du$

$= \frac{u^4}{4} + C = \frac{\tan^4 x}{4} + C$

b) $\int \sec^3 x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int \sec^3 x \tan x \, dx = \int \sec^2 x (\sec x \tan x) \, dx$

$= \int u^2 \, du$

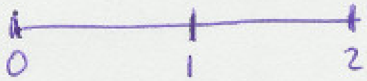
$= \frac{u^3}{3} + C$

$= \frac{\sec^3 x}{3} + C$

4. Consider the definite integral $\int_0^2 \sqrt{1+24x}\,dx$. **[6 marks]**

a) Use the trapezoidal approximation with $n = 2$ subintervals to estimate the value of the integral.

$$\Delta x = \frac{2-0}{2} = 1$$



$$f(x) = \sqrt{1+24x}$$

$$T_2 = \frac{\Delta x}{2}\left(f(0) + 2f(1) + f(2)\right)$$

$$= \frac{1}{2}(1 + 2 \cdot 5 + 7)$$

$$= 9$$

b) Find the exact value of the integral. Is the exact value greater or less than the estimate in part a)?

$u = 1 + 24x \qquad u(0) = 1$

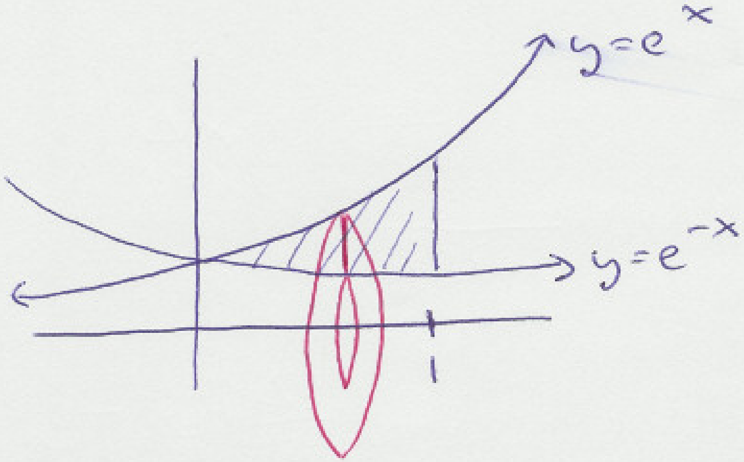
$du = 24\,dx \qquad u(2) = 49$

$$\int_0^2 \sqrt{1+24x}\,dx = \frac{1}{24}\int_1^{49} \sqrt{u}\,du$$

$$= \frac{1}{24}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{49} = \frac{1}{36}\left[7^3 - 1\right] = 9.5$$

The exact value is GREATER than the trapezoidal approximation.

5. Use the method of cross-sections to calculate the volume of the solid generated by rotating the region bounded by $y=e^x$, $y=e^{-x}$ and $x=1$ about the x-axis. **[5 marks]**

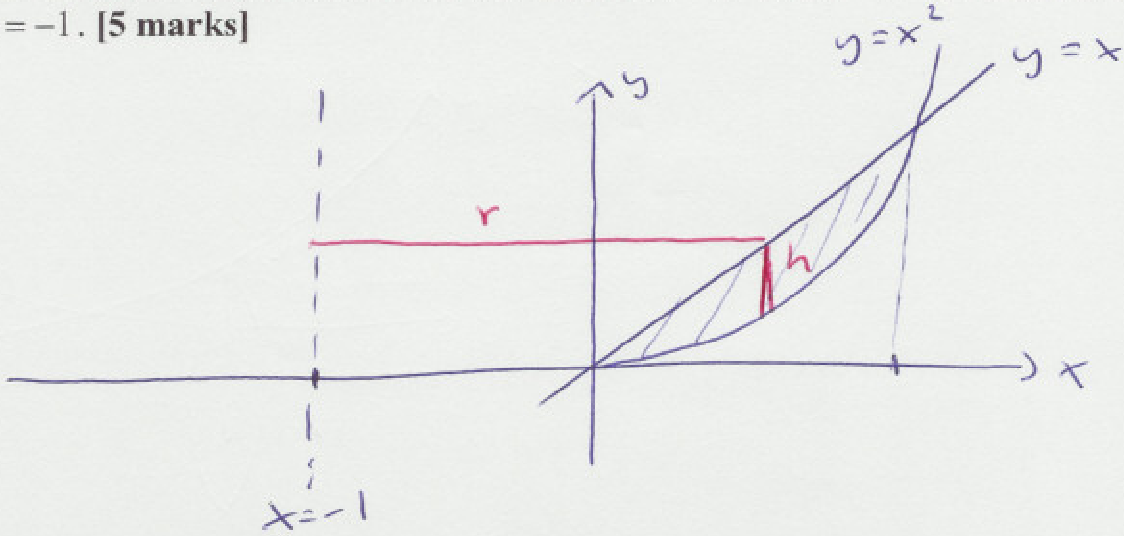


$$V=\int_0^1 \pi\left(e^{2x}-e^{-2x}\right)dx$$

$$=\pi\left[\tfrac{1}{2}e^{2x}+\tfrac{1}{2}e^{-2x}\right]_0^1$$

$$=\frac{\pi}{2}\left[e^2+e^{-2}-2\right]$$

6. Use the method of cylindrical shells to calculate the volume of the solid generated by rotating the region bounded by $y = x^2$ and $y = x$ about the line $x = -1$. **[5 marks]**



$$V = \int_0^1 2\pi(x+1)\left[x - x^2\right] dx$$

$$= 2\pi \int_0^1 (x - x^3)\, dx$$

$$= 2\pi \left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]_0^1$$

$$= \frac{\pi}{2}$$