Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, **Calculus II**
Summer 2006 – Final Exam
August 9th, 2006, 8:30 – 11:30

Last Name (please print): _____

First Name (please print): _____

SFU email ID: _____

Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 14 pages with a total of 10 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **No** calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1 a), b)	**/8**
1 c), d)	**/8**
1 e), f)	**/8**
2	**/8**
3	**/8**
4	**/10**
5	**/8**
6 a), b)	**/8**
6 c), d)	**/8**
7	**/6**
8	**/6**
9	**/6**
10	**/8**
Total	**/100**

1. Evaluate the following, if it is possible: **[4 marks each = 24 marks]**

a) $\int x^5 e^{-x^3}\, dx$

b) $\int_1^5 \sqrt{-x^2+6x-5}\, dx$

c) $\int \frac{(1+x)^{1/2}}{(1-x)^{1/2}}\,dx$

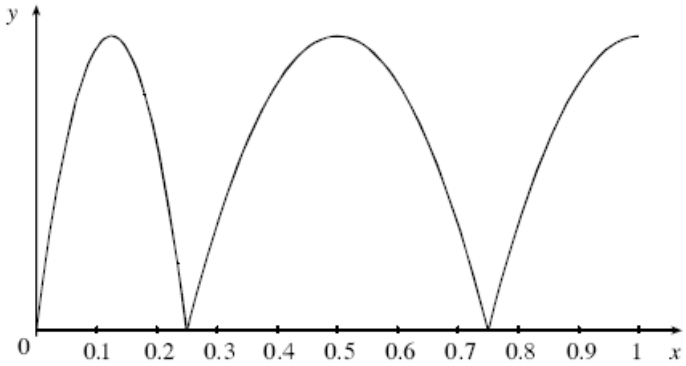
d) $\int \frac{\cos x}{4-\sin^2 x}\,dx$

e) $\int \frac{dx}{\sqrt{1+x^2}}$

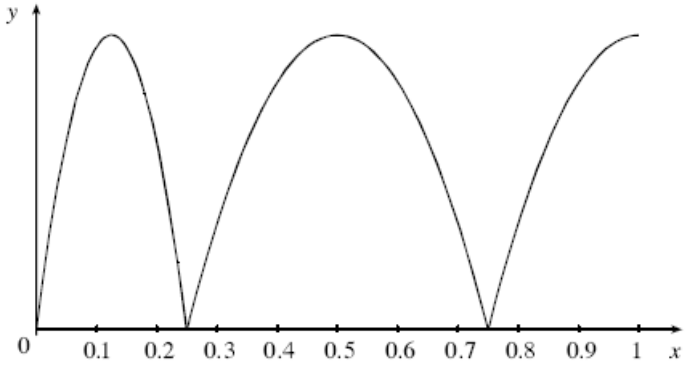
f) $\frac{d}{dx}\int_{\pi}^{\sin x} \ln(t^3-2)\,dt$

2. Let $f(x)$ be the function graphed in sections a) – c). Let $\int_0^1 f(x)\,dx$ be the integral that needs to be approximated. **[8 marks]**

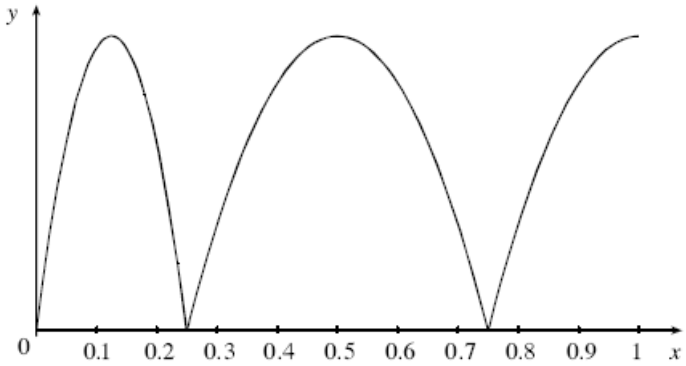
a) Draw the Midpoint Rule with 4 equal subintervals.



b) Draw the Left Endpoint Rule with 4 equal subintervals.

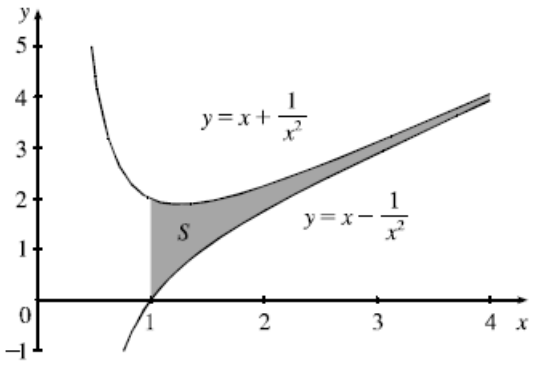


c) Draw the Trapezoidal Rule with 4 equal subintervals.



d) Which of the above three approximation rules gives the best approximation for $\int_0^1 f(x)\,dx$ and why?

3. Consider the region S bounded by the graphs of $y = x + \frac{1}{x^2}$ and $y = x - \frac{1}{x^2}$ for $x \geq 1$, which is shown below. **[8 marks]**

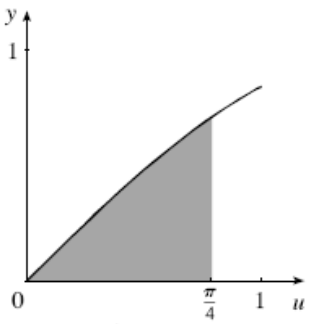


a) Is the area of S finite or infinite? If finite, find the area, if not explain why not.

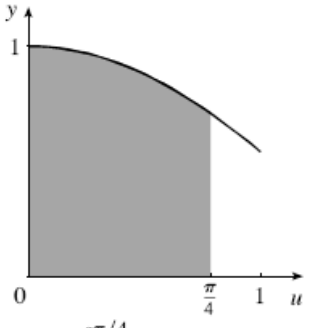
b) Suppose S is rotated about the x-axis generating a solid D. Is the volume of D finite or infinite? If finite, find the volume, if not explain why not.

4. Area and volume:

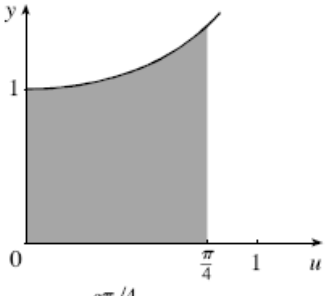
a) What integration bounds *a* and *b* are needed to make $\int_a^b \frac{4dx}{\sqrt{\left(4+x^2\right)^3}}$ equal to one of the areas below? Circle the area. **[4 marks]**



$\int_0^{\pi/4} \sin u\, du$



$\int_0^{\pi/4} \cos u\, du$



$\int_0^{\pi/4} \sec u\, du$

b) Find the area of the region shared by the circle $r = 2$ and the cardioid $r = 2 - 2\cos\theta$. **[6 marks]**

5. Consider the region *R* below between the curves $y = 4x$ and $y = 2x^2 - 2x + 4$. **[8 marks]**

   a) Find the points of intersection.

   b) Graph the region, shade it and provide important labels.

   c) Set up, but do **NOT** evaluate, an integral to find the volume of the solid obtained by rotating *R* about the line $x = -1$. Do **NOT** simplify. Name your method and draw one general disk, washer or shell on the graph above.

6. Test the series for convergence or divergence and name the test(s) used.
[4 marks each = 16 marks]

a) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3}$

b) $\sum_{n=1}^{\infty} \frac{1}{3^n - 5}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+1}$

d) $\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

7. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}}$. **[6 marks]**

8. Find the 2nd degree Taylor polynomial for $f(x) = \sin x^2$ at $a = \sqrt{\pi}$. **[6 marks]**

9. Find the cube roots of $8i$ and write them in the form $a+ib$. **[6 marks]**

10. Choose **one** of the following two problems to solve. Clearly indicate which problem you are solving by **circling** it. **[8 marks]**

a) Find the general solution of $\left(x^2+4\right)\frac{dy}{dx}=xy$.

b) A space module weighs around 5000 N on the surface of Earth. How much work is done in propelling the module to a height of 1300 kilometers above the earth? Use 6400 kilometers as the radius of Earth. Do not consider the effect of air resistance or the weight of the propellant. Recall that Newton's Law of Universal Gravitation states that the force of gravitational attraction between the Earth and other objects is inversely proportional to the distance separating the earth's center from the object's center.