Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, Calculus II
Spring 2006 – Final Exam
April 15th, 2006, 8:30 – 11:30

Last Name (please print): _____

First Name (please print): _____

SFU email ID: _____

Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 14 pages with a total of 10 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1 a), b)	/8
1 c), d)	/8
1 e), f)	/8
2	/7
3	/15
4	/6
5	/6
6 a), b)	/8
6 c), d)	/8
7	/6
8	/6
9	/6
10	/8
Total	/100

1. Evaluate the following, if it is possible: **[4 marks each = 24 marks]**

a) $\int x^2 (\ln x)^2 \, dx$

$u = (\ln x)^2 \qquad dv = x^2 \, dx$

$du = 2 \ln x \cdot \frac{1}{x} \, dx \qquad v = \frac{x^3}{3}$

$= \frac{1}{3} x^3 (\ln x)^2 - \frac{2}{3} \int x^2 \ln x \, dx \qquad u = \ln x$

$du = \frac{1}{x} dx$

$= \frac{1}{3} x^3 (\ln x)^2 - \frac{2}{9} x^3 \ln x + \frac{2}{9} \int x^2 \, dx$

$= \frac{1}{3} x^3 (\ln x)^2 - \frac{2}{9} x^3 \ln x + \frac{2}{27} x^3 + C$

b) $\int_0^{\frac{\pi}{2}} \cos^3 x \sin 2x \, dx$

$= \int_0^{\frac{\pi}{2}} \cos^3 x \; 2 \sin x \cos x \, dx$

$= \int_0^{\frac{\pi}{2}} 2 \cos^4 x \sin x \, dx$

$= -\frac{2}{5} \cos^5 x \Big]_0^{\frac{\pi}{2}}$

$= \left(-\frac{2}{5} \cos^5 \left(\frac{\pi}{2}\right) + \frac{2}{5} \cos^5 (0) \right)$

$= \frac{2}{5}$

c) $\int \frac{3}{x^{-1/2}\left(x^{3/2}-x^{1/2}\right)}\,dx$

$= \int \frac{3}{x-1}\,dx$

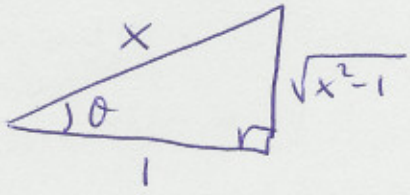
$= 3\ln|x-1| + C$

d) $\int \frac{\sqrt{x^2-1}}{x}\,dx$

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$= \int \frac{\tan\theta\ \sec\theta\tan\theta\ d\theta}{\sec\theta}$



$= \int \tan^2\theta\,d\theta$

$= \int \sec^2\theta - 1\,d\theta$

$= \tan\theta - \theta + C$

$= \sqrt{x^2-1} - \sec^{-1}(x) + C$

e) $\int_0^3 \frac{dx}{x^2-x-2}$ ← undefined for $x=2$

$= \int_0^2 \frac{dx}{x^2-x-2} + \int_2^3 \frac{dx}{x^2-x-2}$

Now, $\int \frac{dx}{x^2-x-2} = \int \frac{dx}{(x-2)(x+1)} = \int \frac{\frac{1}{3}}{x-2} - \frac{\frac{1}{3}}{x+1}\, dx$

$= \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + C$

So, $\int_0^2 \frac{dx}{x^2-x-2} = \frac{1}{3}\lim_{t\to 2^-} (\ln|t-2| - \ln|t+1|) - \frac{1}{3}(\ln 2) =$

Hence, $\int_0^3 \frac{dx}{x^2-x-2}$ is divergent.

f) $\frac{d}{dx}\int_e^{\ln x} \sin(t^2+1)\, dt$

$= \sin((\ln x)^2+1) \cdot \frac{1}{x}$

2. Choose **one** of the following three methods to do part a) ~~and b)~~. Clearly indicate which method you are using by **circling** it. **[7 marks]**

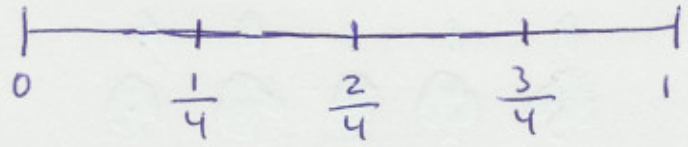
Midpoint Rule

Trapezoidal Rule

Simpson's Rule

$f(x) = x^4 + 1$

a) Approximate $\int_0^1 x^4 + 1\,dx$ with $n = 4$. Round your answer to four decimal places.



$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

Midpoint Rule

$$\int_0^1 x^4+1\,dx \approx \frac{1}{4}\left(f\left(\frac{1}{8}\right) + f\left(\frac{3}{8}\right) + f\left(\frac{5}{8}\right) + f\left(\frac{7}{8}\right)\right)$$

$$\approx 1.1897$$

Trapezoidal Rule

$$\int_0^1 x^4+1\,dx \approx \frac{\frac{1}{4}}{2}\left[f(0) + 2f\left(\frac{1}{4}\right) + 2f\left(\frac{2}{4}\right) + 2f\left(\frac{3}{4}\right) + f(1)\right]$$

$$\approx 1.2207$$

Simpson's Rule

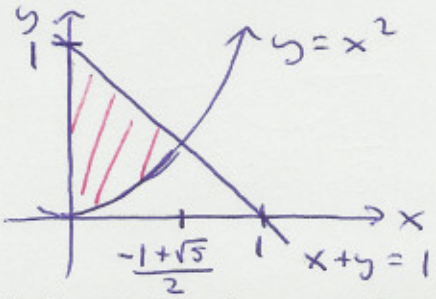
$$\int_0^1 x^4+1\,dx \approx \frac{\frac{1}{4}}{3}\left[f(0) + 4f\left(\frac{1}{4}\right) + 2f\left(\frac{2}{4}\right) + 4f\left(\frac{3}{4}\right) + f(1)\right]$$

$$\approx 1.2005$$

the y-axis

3. A region R is bounded by the graphs of $y = x^2$, $x + y = 1$ and ~~$x \neq 0$~~ in the first quadrant. **[3 marks each = 15 marks]**

a) Sketch the region R and shade it. Clearly indicate all pertinent points.



$$x^2 + x - 1 = 0$$

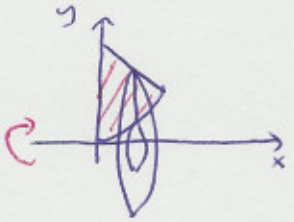
$$x = \frac{-1 \pm \sqrt{5}}{2}$$

b) Set up, but do not evaluate, an integral to find the following:

i) The area of R.

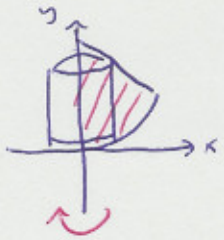
$$A = \int_0^{\frac{-1+\sqrt{5}}{2}} (1-x) - x^2 \, dx$$

ii) The volume of the solid obtained by rotating R about the x-axis.



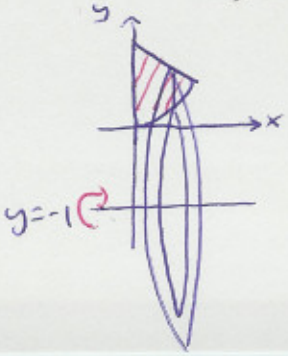
$$V = \int_0^{\frac{-1+\sqrt{5}}{2}} \pi \left[(1-x)^2 - (x^2)^2 \right] dx$$

iii) The volume of the solid obtained by rotating R about the y-axis.



$$V = \int_0^{\frac{-1+\sqrt{5}}{2}} 2\pi x \left[(1-x) - x^2 \right] dx$$

iv) The volume of the solid obtained by rotating R about the line $y = -1$.



$$V = \int_0^{\frac{-1+\sqrt{5}}{2}} \pi \left[(1+(1-x))^2 - (1+x^2)^2 \right] dx$$