Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 152-3, **Calculus II**
Spring 2006 – Final Exam
April 15th, 2006, 8:30 – 11:30

Last Name (please print): _____

First Name (please print): _____

SFU email ID: _____

Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 14 pages with a total of 10 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1 a), b)	**/8**
1 c), d)	**/8**
1 e), f)	**/8**
2	**/7**
3	**/15**
4	**/6**
5	**/6**
6 a), b)	**/8**
6 c), d)	**/8**
7	**/6**
8	**/6**
9	**/6**
10	**/8**
Total	**/100**

1. Evaluate the following, if it is possible: **[4 marks each = 24 marks]**

a) $\int x^2 \left(\ln x\right)^2 \, dx$

b) $\int_0^{\frac{\pi}{2}} \cos^3 x \sin 2x \, dx$

c) $\int \frac{3}{x^{-1/2}\left(x^{3/2} - x^{1/2}\right)}\, dx$

d) $\int \frac{\sqrt{x^2 - 1}}{x}\, dx$

e) $\int_0^3 \frac{dx}{x^2 - x - 2}$

f) $\frac{d}{dx}\int_e^{\ln x} \sin(t^2 + 1)\, dt$

2. Choose **one** of the following three methods to do parts a). Clearly indicate which method you are using by **circling** it. **[7 marks]**

Midpoint Rule

Trapezoidal Rule

Simpson's Rule

a) Approximate $\int_0^1 x^4 + 1 dx$ with $n = 4$. Round your answer to four decimal places.

3. A region *R* is bounded by the graphs of $y = x^2$, $x + y = 1$ and the *y*-axis in the first quadrant. **[3 marks each = 15 marks]**

   a) Sketch the region *R* and shade it. Clearly indicate all pertinent points.

   b) Set up, but do not evaluate, an integral to find the following:

      i) The area of *R*.

      ii) The volume of the solid obtained by rotating *R* about the *x*-axis.

      iii) The volume of the solid obtained by rotating *R* about the *y*-axis.

      iv) The volume of the solid obtained by rotating *R* about the line $y = -1$.

4. Find the area of the region enclosed by the 3-leaved rose $r = 4\sin(3\theta)$.
   [6 marks]

5. Consider the formula $\int xf'(x)\,dx = xf(x) - \int f(x)\,dx$, where f is a differentiable function. **[6 marks]**

   a) Show that the formula is valid.

   b) Use the formula to compute $\int \frac{x}{\sqrt[3]{x-1}}\,dx$.

   c) Suppose that $y = f(x)$ is differentiable, $f(2) = -2$, $f(6) = -6$ and $\int_2^6 f(x)\,dx = -10$. Compute $\int_2^6 xf'(x)\,dx$.

6. Test the series for convergence or divergence and name the test(s) used.
[4 marks each = 16 marks]

a) $\sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}}$

b) $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$

c) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$

d) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

7. Find the interval of convergence for the power series $\sum_{n=1}^{\infty}(-1)^n\left[\frac{x^n}{n^5}\right]$.

[6 marks]

8. Find the 4^{th} degree Taylor polynomial for $f(x) = e^{x^2}$ at $a = 1$. **[6 marks]**

9. Find the cube roots of $1+i$ and sketch the roots in the complex plane.
[6 marks]

10. Choose **one** of the following three problems to solve. Clearly indicate which problem you are solving by **circling** it. **[8 marks]**

a) The force of an object is proportional to the square root of its distance from a set rest position. When a certain object is 4 feet from rest, there is a force of 8 pounds on it, pushing it back toward the rest position. What is the work required to displace the object by 16 feet from its rest position.

b) Consider the curve $x = e^{t-5}\cos t$, $y = e^{t-5}\sin t$, $0 \le t \le 2\pi$.
   i) Find the values of *t* where the line tangent to the curve is vertical.
   ii) Find the values of *t* where the slope of the line tangent to the curve is -1.

c) A tank contains 100 liter of pure water. Brine that contains 0.1 kilogram of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?