Simon Fraser University
Department of Mathematics
Burnaby and Surrey Campus

MATH 152-3, Fall 2005
Final Examination
December 7th, 2005

Last Name (please print): ____________________

First Name (please print): ____________________

Student Number: ____________________

Instructor: B. Kadonoff I. Mercer

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 15 pages with a total of 13 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1	**/24**
2	**/6**
3	**/5**
4	**/9**
5	**/6**
6	**/6**
7	**/6**
8	**/6**
9	**/8**
10	**/6**
11	**/6**
12	**/6**
13	**/6**
Total	**/100**

1. Evaluate the following: **[4 marks each = 24 marks]**

a) $\int \frac{1}{\sqrt{\left(x^2+9\right)^3}}\,dx$

b) $\int_1^e \frac{\ln x}{x}\,dx$

c) $\int \cos^2 5x \, dx$

d) $\int x^3 \ln x \, dx$

e) $\int x\sec(x^2)\tan(x^2)\,dx$

f) $\int \frac{3x+1}{x(x+1)}\,dx$

2. Determine if the improper integral $\int_0^{\infty} e^{-(x+1)}\,dx$ converges or diverges. Compute its value if it converges. **[6 marks]**

3. Let R be the region bounded by the graphs $y = \frac{1}{4}x^2$ and $x - 2y + 4 = 0$. **[5 marks]**

   a) Sketch the region R.

   b) Find the area R.

4. Let R be the region bounded by the graphs $y^2 = x$ and $y = x$. Set up, but do **not** evaluate, the integral to find the **volume** of the solid of revolution described below. Provide a sketch with each part.
[9 marks]

a) R is revolved about the x-axis.

b) R is revolved about the y-axis.

c) R is revolved about the line $y = 2$.

5. Consider the integral $I = \int_0^{2\pi} x \sin x \, dx$. Approximate I with $n=4$ by the following methods: **[3 marks each = 6 marks]**

   a) Riemann sum with left end-points.

   b) Trapezoid rule.

6. If it takes 200 Newton-metres of work to stretch a certain spring 1 meter beyond its natural length, how much work is required to stretch that same spring ½ meter beyond its natural length? **[6 marks]**

7. A metal spatula is being used in cooking. Its blade is at a temperature of $200^\circ\mathrm{C}$. The blade is plunged into a sink full of water whose temperature is $40^\circ\mathrm{C}$. After 2 seconds in the water the blade has cooled to $130^\circ\mathrm{C}$. How long will it take the blade to cool to $60^\circ\mathrm{C}$ so that the cook can safely handle the blade. **[6 marks]**

8. Find the area inside one half of the lemniscate $r^2 = \cos(2\theta)$. **[6 marks]**

9. Test the series for convergence or divergence and name the test(s) used.
[4 marks each = 8 marks]

a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt[3]{n^2+4}}{n^3+5}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{n3^n}$

10. Find the interval of convergence for the power series $\sum_{n=0}^{\infty}(-1)^n\left[\frac{x^{2n+1}}{\sqrt{2n+1}}\right]$.

[6 marks]

11. Find the Maclaurin series for $f(x) = \cosh x$. **[6 marks]**

12. Answer the complex number problems below. **[6 marks]**

a) Write $-\sqrt{3}+i$ in polar form.

b) Find all the roots of $z^4 = -16$.

13. Determine whether $\int_0^\infty \sin(x^2)\,dx$ is absolutely convergent, conditionally convergent or divergent. Use $a_n = \int_{\sqrt{n\pi}}^{\sqrt{(n+1)\pi}} \sin(x^2)\,dx, n \geq 0$ and the function $f(x) = \sin(x^2)$ graphed below. Name the test(s) used. **[6 marks]**