MATH 152-D200 Instructor: R. Pyke

Midterm 2, March 10, 2008

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1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.
5. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

Question	Score	Max
1		23
2		8
3		4
4		6
5		6
Total		47

There is a page of formulae on the last page and a table of integrals. Except for question 1, you can use the table of integrals to evaluate integrals that arise in the remaining problems.

(1) [Marks: 23] Find the indicated definite or indefinite integral. Remember, these questions you are being asked to show that you understand the techniques we've learned in class, so do not just write down an answer because you remember it.

(a) $\displaystyle\int \frac{3\sqrt{1+\tan^{-1} x}}{2+2x^2}\,dx$

(b) $\displaystyle\int \frac{(\ln 3x)^2}{x^4}\,dx$

(c) $\displaystyle\int_{\pi/4}^{3\pi/4} 3\csc^4 z \cot^6 z \, dz$

(d) $\displaystyle\int x\,\sqrt{x^2+3x}\;\, dx$ (hint: Complete the square first)

For this problem you are allowed to use the integral tables at the back.

(e) $\displaystyle\int \frac{2x}{(x-1)(x^2+2x+5)}\,dx$

(f) $\displaystyle\int_{-5}^{5} 7\,x^{13}\,(\sin 2x)^{12}\cos 5x\,dx$

For the remaining problems (2 - 5), you can use the table of integrals at the back to evaluate integrals that arise in the problems.

(2) [Marks: 8] (a) Evaluate

$$\int_0^2 \frac{\ln x}{\sqrt{2x}}\,dx$$

(b) Is the following integral convergent or divergent? Explain your answer

$$\int_8^\infty \frac{x+1}{\sqrt{x^4 - x}}\, dx$$

(Note you are **not** asked to evaluate the integral!)

(3) [Marks: 4] How large should n be to guarantee that the Trapeziodal Rule approximation to $\int_0^1 e^{2x^3}\, dx$ is accurate to within 0.001?

(4) [Marks: 6] Set up, **but do not evaluate**, the integral that gives the surface area of the surface of revolution obtained by rotating the graph $y = \ln x + 1, \quad 1 \leq x \leq 4$, about

(a) the x-axis

(b) the y-axis

(5) [Marks: 6] Determine the length of the boundary of the (finite) region between the graphs $y = x^2 - 1$ and $y = 1 - x^2$.

Formula Sheet

$$
\begin{aligned}
1+\tan^2 x &= \sec^2 x \\
1+\cot^2 x &= \csc^2 x \\
\sin(x+y) &= \sin x\cos y+\cos x\sin y \\
\sin(x-y) &= \sin x\cos y-\cos x\sin y \\
\cos(x+y) &= \cos x\cos y-\sin x\sin y \\
\cos(x-y) &= \cos x\cos y+\sin x\sin y \\
\tan(x+y) &= \frac{\tan x+\tan y}{1-\tan x\tan y} \\
\tan(x-y) &= \frac{\tan x-\tan y}{1+\tan x\tan y} \\
\sin 2x &= 2\sin x\cos x \\
\cos 2x &= \cos^2 x-\sin^2 x \;=\; 2\cos^2 x-1 \;=\; 1-2\sin^2 x \\
\tan 2x &= \frac{2\tan x}{1-\tan^2 x} \\
\sin^2 x &= \frac{1-\cos 2x}{2} \\
\cos^2 x &= \frac{1+\cos 2x}{2}
\end{aligned}
$$

$$
\begin{aligned}
\int_a^b f(x)\,dx &\approx M_n \;=\; \Delta x[f(\bar{x}_1)+f(\bar{x}_2)+\cdots+f(\bar{x}_n)] \\
\int_a^b f(x)\,dx &\approx T_n \;=\; \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)] \\
\int_a^b f(x)\,dx &\approx S_n \;=\; \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]
\end{aligned}
$$

$$
\begin{aligned}
|E_T| &\le \frac{K(b-a)^3}{12n^2} \\
|E_M| &\le \frac{K(b-a)^3}{24n^2} \\
|E_S| &\le \frac{K(b-a)^5}{180n^4}
\end{aligned}
$$

Table of Integrals $\longrightarrow$

Table of Integrals
(add constant C to the right side)

$$
\begin{aligned}
\int \tan x\,dx &= \ln|\sec x| \\
\int \cot x\,dx &= \ln|\sin x| \\
\int \sec x\,dx &= \ln|\sec x + \tan x| \\
\int \csc x\,dx &= \ln|\csc x - \cot x| \\
\int \sin^n x\,dx &= -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx \\
\int \cos^n\ dx &= \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx \\
\int \sec^n x, dx &= \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx \quad (n \neq 1) \\
\int \tan^n x\,dx &= \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx \quad (n \neq 1) \\
\int x^n e^x\,dx &= x^n e^x - n\int x^{n-1} e^x\,dx \\
\int (\ln x)^n\,dx &= x(\ln x)^n - n\int (\ln x)^{n-1}\,dx
\end{aligned}
$$