Answer Key

Math 151-D1 Calculus I
Midterm 2, March 8
Instructor: Matt DeVos

First Name (please print): ______________________

Last Name (please print): ______________________

SFU email ID: ______________________

Signature: ______________________

1. DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
2. Fill in the above box.
3. Once the exam begins, check that your exam has 6 questions and 7 pages.
4. Only the usual writing instruments, this booklet, and a scientific calculators are allowed. **No graphing or programmable calculators are permitted.**
5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.
6. You may use the back of the previous page for rough work or if you run out of space.
7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Problem	Score	Value
1		4
2		4
3		2
4		4
5		4
6		6
Total:		24

Problem 1. *(4 points)* Mark each statement as true (**T**) or false (**F**)

F If $f'(x) > 0$ for every x, then $f(x)$ has at most one real root.

T $\sinh(-x) = -\sinh(x)$

F If $f(x)$ is a continuous function, then $f(x)$ has a global maximum.

F If c is a local minimum of the function $f(x)$, then $f'(c) = 0$.

Problem 2. *(4 points)* Find y' for each of the following functions.

(i) *(2 points)* $y = \ln(\cosh x)$

$$y' = \frac{(\cosh x)'}{\cosh x} = \frac{\sinh x}{\cosh x} = \tanh x$$

(ii) *(2 points)* $y = x^{(x^2+1)}$

$$\ln y = \ln\left(x^{(x^2+1)}\right)$$

$$\ln y = (x^2+1)\ln(x)$$

$$\frac{y'}{y} = 2x \cdot \ln x + (x^2+1)(1/x)$$

$$y' = x^{(x^2+1)}\left(2x \cdot \ln x + (x^2+1)(1/x)\right)$$

Problem 3. *(2 points)* Find $\frac{d^n y}{dx^n}$ for the function $y = e^{-3x}$.

$$y = e^{-3x}$$

$$y' = (-3)\, e^{-3x}$$

$$y'' = (-3)(-3)\, e^{-3x}$$

$$y''' = (-3)(-3)(-3)\, e^{-3x}$$

$$\frac{d^n y}{dx^n} = (-3)^n \cdot e^{-3x}$$

Problem 4. *(4 points)* Verify that the function $f(x) = x^3 + 4x$ on the interval $[-1, 2]$ satisfies the hypothesis of the Mean Value Theorem. Find all points c which satisfy the conclusion.

$f(x)$ is a polynomial so it is continuous on $[-1,2]$ and diff. on $(-1,2)$

(in fact, $f(x)$ is cont. and diff on all of $\mathbb{R}$)

By the M.V.T. there exist points c in $(-1,2)$

so that $f'(c) = \dfrac{f(2) - f(-1)}{2-(-1)} = \dfrac{16-(-5)}{3} = 7$

to find all such values..

$7 = f'(x) = 3x^2 + 4$

so $3x^2 = 3$

$x = \pm 1$

So $c = 1$ is the only point in $(-1,2)$ which satisfies the conclusion.

Problem 5. *(4 points)* Find the maximum and minimum of the function $f(x) = \frac{x}{x^2+1}$ on the interval $[0, 2]$.

Using the closed interval method

endpoints

$f(0) = 0$

$f(2) = 2/5$

---

to find the critical points

$$0 = f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{(1-x)(1+}{(x^2+1)^{}}$$

So the only critical point in $(0,2)$ is $x = 1$

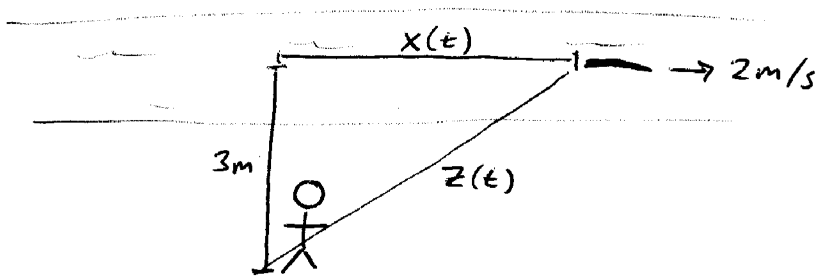
$f(1) = \frac{1}{2}$

---

max (obtained at $x=1$) value $\frac{1}{2}$

min (obtained at $x=0$) value 0

Problem 6. *(6 points)* A man facing North sees a river which flows East. He tosses a stick into the water exactly $3m$ North of where he stands. The river carries the stick East at a constant rate of $2m/s$. How fast is the stick moving away from the man after 2 seconds?



$x(t)$ = distance of stick from its starting point (so $\frac{dx}{dt} = 2$)

$z(t)$ = distance of stick from man

$$z^2 = x^2 + 9$$

$$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt}$$

at time $t = 2$ $x = 4$ and $z^2 = 16 + 9$ so $z = 5$

at this time

$$2 \cdot 5 \cdot \frac{dz}{dt} = 2 \cdot 4 \cdot 2$$

$$\frac{dz}{dt} = \frac{16}{10} = \frac{8}{5} \text{ m/s}$$