

Simon Fraser University  
Department of Mathematics  
Burnaby Campus

**MATH 151-3**, Spring 2005  
Midterm 2  
March 9<sup>th</sup>, 2005, 8:30 – 9:20 am

Last Name (please print): \_\_\_\_\_

First Name (please print): \_\_\_\_\_

Student Number: \_\_\_\_\_

**Instructions:**

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 8 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. **SHOW ALL YOUR WORK!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific calculators are allowed (basic math/stat functions + memory key).
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your best!

<b>Do not write in this table!</b>	
<b>Question</b>	<b>Marks</b>
1	/8
2	/6
3	/5
4	/8
5	/7
6	/6
<b>Total</b>	<b>/40</b>

1.

- (a) [3 marks] Let  $f(t) = \ln \frac{\sqrt{4t-7}}{3t-2}$ . Apply laws of logarithms to simplify the function  $f(t)$  before finding  $f'(t)$ . Do NOT simplify your answer.

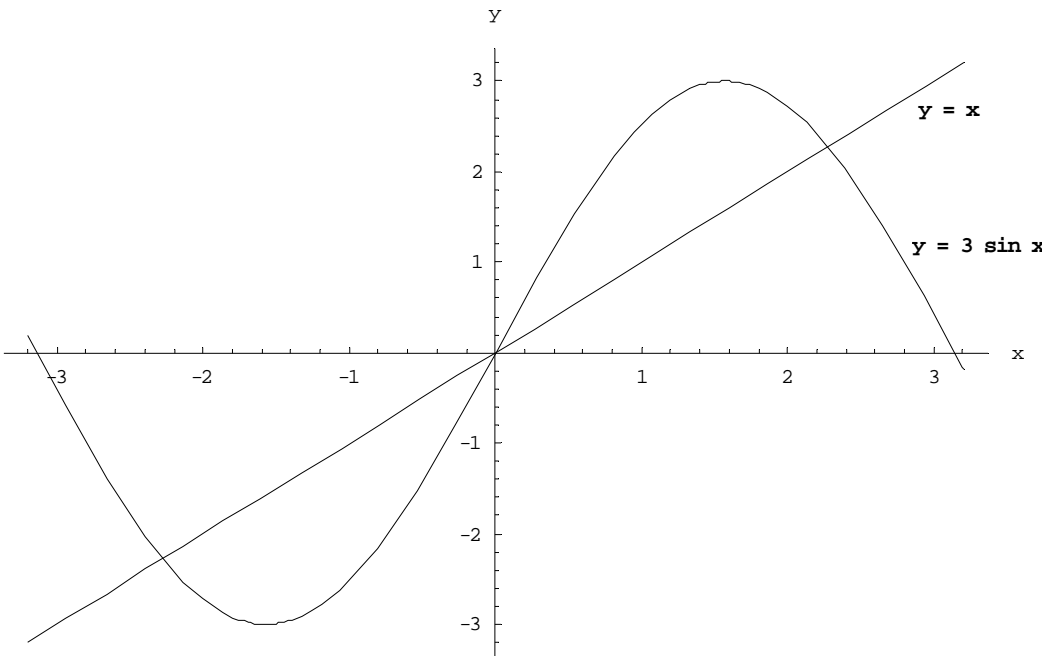
- (b) [5 marks] Find  $\frac{dy}{dx}$  in terms of  $x$  only, given that  $y = (\ln x)^{\sin x}$ . Do NOT simplify your answer.

2. [6 marks] Find the maximum possible area of a rectangle whose diagonals each have length  $\sqrt{32}$  cm.

3. [5 marks] The radius  $r$  of a sphere and the edge length  $x$  of a cube are changing in such a way that their combined volume is decreasing at a rate of  $18\pi \text{ cm}^3/\text{s}$  and the edge length  $x$  is increasing at the rate of  $\pi \text{ cm/s}$ . When  $r = 3 \text{ cm}$  and  $x = 3 \text{ cm}$ , is the radius  $r$  increasing or decreasing and at what rate? (The volume of the sphere is  $\frac{4}{3}\pi r^3$ .)

4.

- (a) [2 marks] The figure below shows the graphs of  $y = x$  and  $y = 3 \sin x$ . Mark on the  $x$ -axis the approximate location of each solution to the equation  $x = 3 \sin x$ , and write down an interval containing the largest solution.



- (b) [5 marks] Use Newton's Method to approximate the largest solution of  $x = 3 \sin x$  to two decimal places.

- (c) [1 mark] If the value of the solution determined in part (c) is  $x = a$ , write down all the other solutions to  $x = 3 \sin x$  without carrying out any further calculations.

5.

(a) [2 marks] What does it mean for a function  $f(x)$  to be *increasing* on an open interval  $(a, b)$ ?

(b) [5 marks] Find and classify the critical points (minimum or maximum, local or global, or not an extremum) of  $f(x) = \sin x - x \cos x$  on the open interval  $(-5, 5)$ .

6.

- (a) [2 marks] Give an approximate expression for  $\Delta y$  (the change in  $y$ ) when  $x$  changes by an amount  $\Delta x$ , where  $y$  is a function of  $x$ .

- (b) [4 marks] A pendulum has length  $x$  cm and a period of oscillation  $T$  seconds, where

$T = \frac{2\pi}{\sqrt{10}} \sqrt{x}$ . Use your answer to part (a) to estimate to 3 decimal places the maximum error in the calculated value of  $T$  if  $x$  is measured to be 64 cm to within  $\pm 0.5$  cm.