Simon Fraser University
Math 151-3, Spring 2004
Test 2 Solution (Grey version)

1. Find the following limits if they exist. Show and justify your work.

   (a) [4 marks] $\displaystyle\lim_{x\to 4^+} \frac{x-4}{\sqrt{x}-2}$

   Answer $\displaystyle\lim_{x\to 4^+} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4^+} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \lim_{x\to 4^+} (\sqrt{x}+2) = 4$

   (b) [4 marks] $\displaystyle\lim_{x\to\infty} \frac{\ln(5+x)}{x+\ln x}$

   Answer Since $\displaystyle\lim_{x\to\infty} \ln(5+x) = \infty = \lim_{x\to\infty}(x+\ln x)$, L'Hopital's Rule can be used:

$$\lim_{x\to\infty} \frac{\ln(5+x)}{x+\ln x} = \lim_{x\to\infty} \frac{1/(5+x)}{1+1/x} = \frac{0}{1+0} = 0.$$

2. (a) [4 marks] Find $f'(t)$ if $f(t) = \dfrac{t^2-2t+7}{e^{3t}}$. Do not simplify your answer.

   Answer By the Quotient Rule,

$$f'(t) = \frac{(2t-2)e^{3t} - \left(t^2-2t+7\right)\cdot 3e^{3t}}{\left(e^{3t}\right)^2} = \frac{-3t^2+8t-23}{e^{3t}}$$

   (b) [4 marks] Differentiate $g(x) = x^x\sqrt{2+\sin x}$. Express your answer as a function of x.

   Answer Since $\ln g(x) = \ln\left(x^x\sqrt{2+\sin x}\right) = x\ln x + \frac{1}{2}\ln(2+\sin x)$,

$$\frac{d}{dx}\ln g(x) = \frac{d}{dx}\left[x\ln x + \frac{1}{2}\ln(2+\sin x)\right] \Longrightarrow \frac{g'(x)}{g(x)} = \ln x + x\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{\cos x}{2+\sin x}$$
$$\Longrightarrow g'(x) = g(x)\left(\ln x + 1 + \frac{1}{2}\cdot\frac{\cos x}{2+\sin x}\right) = x^x\sqrt{2+\sin x}\left(\ln x + 1 + \frac{1}{2}\cdot\frac{\cos x}{2+\sin x}\right)$$

3. (a) [4 marks] Let $f(x) = 2x^3+5x^2-12x-11$. The equation

$$f(x) = 0$$

   has a solution near $x_1 = 2$. Use Newton's Method to find a better approximation x_2 of the solution to this equation. Express your answer as a fraction.

   Answer $\displaystyle x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2(2)^3+5(2)^2-12(2)-11}{6(2)^2+10(2)-12} = \frac{63}{32}$

   (b) [4 marks] Find $\dfrac{dy}{dx}$ if $x^3 = 5x^2y+2y^3$.

   Answer

$$\frac{d}{dx}x^3 = 5\frac{d}{dx}x^2y + 2\frac{d}{dx}y^3 \Longrightarrow 3x^2 = 5\left(2xy + x^2\frac{dy}{dx}\right) + 6y^2\frac{dy}{dx} \Longrightarrow \frac{dy}{dx} = \frac{3x^2-10xy}{5x^2+6y^2}$$

4. Let P be a point on the curve of the function $y = \sqrt{5x}$, and let s be the distance between P and the origin $(0,0)$.

   (a) [2 marks] Express s as a function of the x-coordinate x of P.

   Answer $s = \sqrt{x^2 + 5x}, \quad x \geq 0$

   (b) [4 marks] Suppose the x-coordinate of P increases at a constant rate of $\sqrt{14}$ units per second. How fast is s changing when the x-coordinate of P is 2 units?

   Answer Since
$$\frac{ds}{dt} = \frac{ds}{dx}\frac{dx}{dt} = \frac{2x+5}{2\sqrt{x^2+5x}} \cdot \sqrt{14},$$
$$\left.\frac{ds}{dt}\right|_{x=2} = \frac{2\,(2)+5}{2\sqrt{2^2+5\,(2)}} \cdot \sqrt{14} = \frac{9}{2} \text{ units per second.}$$

5. For this question, let $f(x) = x^{1/3}(x-3)$.

   (a) [4 marks] Find all critical points of f.

   Answer Since $f(x)$ is continuous everywhere, the critical points of $f(x)$ will be all the points where $f'(x)$ is zero or undefined. Since
$$f'(x) = \left(x^{4/3} - 3x^{1/3}\right)' = \frac{4}{3}x^{1/3} - x^{-2/3} = \frac{4x-3}{3x^{2/3}},$$
The critical points of $f(x)$ are $x = 0$ and $3/4$.

   (b) [4 marks] Determine the open intervals on which $f(x)$ is increasing as well as those on which $f(x)$ is decreasing.

   Answer First Derivative Test:

$f'(x)$	$-$		$-$		$+$
$f(x)$	$\searrow$		$\searrow$		$\nearrow$
x		0		$3/4$	

   Hence $f(x)$ is increasing on $(3/4, \infty)$, and $f(x)$ is decreasing on $(-\infty, 0)$ and $(0, 3/4)$.

   (c) [2 marks] Based on your results in Parts (a) and (b), classify each of the critical points of $f(x)$ as local maximum or minimum, absolute maximum or minimum, or not an extremum. (For example, "$f(x)$ has an absolute maximum at $x = 16$").

   Answer $f(x)$ has no extremum at $x = 0$, and $f(x)$ has an absolute minimum at $x = 3/4$.