MATH 151
Midterm 2, November 2, 2005

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1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Circle your instructor. If you don't, you lose a mark.
3. This test is comprised of 5 pages.
4. Once the test begins, please check that all pages are intact.
5. Do ALL questions.
6. Clearly explain your answer. No credit will be given for just writing down the answer.
7. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
8. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.

Question	Score	Max
1		7
2		4
3		9
4		6
5		4
Total		30

1) Find the indicated derivatives of the following functions. You do *not* need to simplify your answers.

(1a) (4 marks) y' *and* y'' where $y = \cos(e^{2x})$

Solution:

$$\begin{aligned} y' &= -2e^{2x}\sin(e^{2x}) \\ y'' &= -4e^{2x}\sin(e^{2x}) - 4e^{4x}\cos(e^{2x}) \end{aligned}$$

(1b) (3 marks) $g'(t)$ where $g(t) = t^{\sqrt{t\ln t}}, \quad t > 1$

Solution:

Let $h(t) = \ln(g(t)) = \sqrt{t\ln t}\ln t$. Then $h'(t) = \dfrac{(\ln t + 1)(\ln t)}{2\sqrt{t\ln t}} + \dfrac{\sqrt{t\ln t}}{t}$. Thus, $g'(t) = g(t)h'(t)$.

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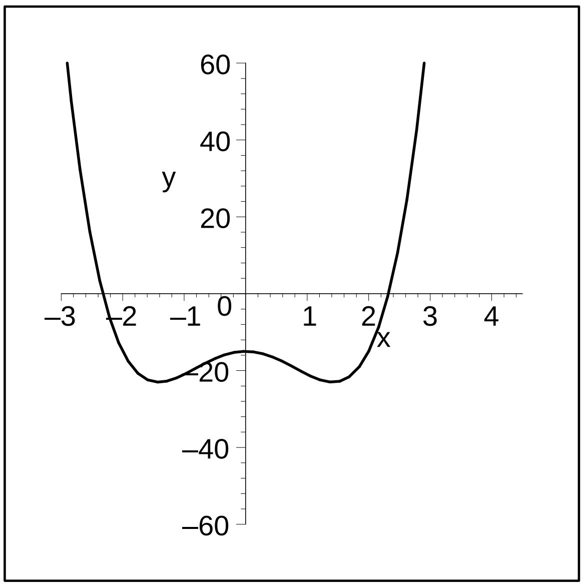
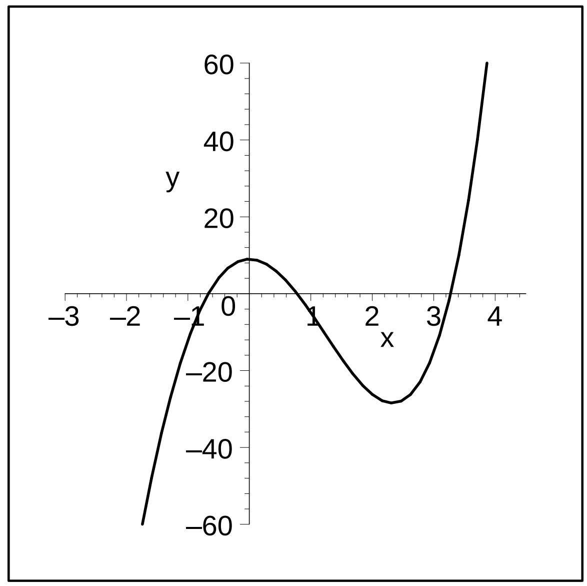
2) (4 marks) Below are the graphs of four functions. Among them are the graphs of $f(x)$, $f'(x)$ and $f''(x)$. Determine which of the graphs are the graphs of $f(x)$, $f'(x)$ and $f''(x)$ and fill in the spaces below with your choice of a, b, c or d. (No explanation is required.)

Answer

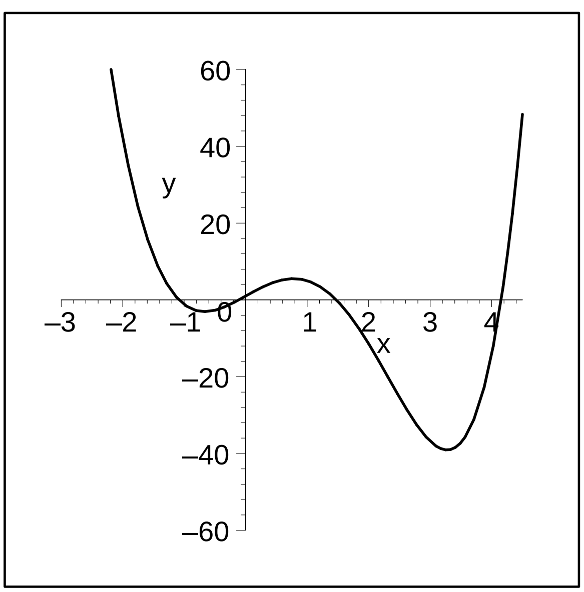
f: c
f': b
f'': d



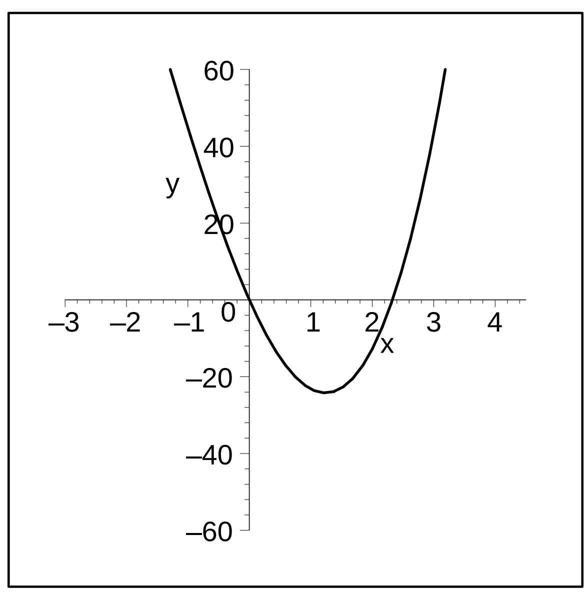
a



b



c



d

3) Consider the curve defined by $x^2 + 2x + 2y^2 = xy + y$.

(3a) (1 mark) Show that $(0, 0)$ lies on the curve.

Solution:

If we fill in $(x, y) = (0, 0)$ in the equation then we get $0^2 + 2 \cdot 0 + 2 \cdot 0^2 = 0 \cdot 0 + 0$, which is true, so clearly $(0, 0)$ satisfies the equation.

(3b) (2 marks) Use implicit differentiation to find y'.

Solution:

We implicitly consider y to be a function of x. Therefore,

$$x^2 + 2x + 2y^2 = xy + y$$

is an equality between functions in x, so if we take derivatives with respect to x, equality should still hold:

$$2x + 2 + 4y\frac{dy}{dx} = x\frac{dy}{dx} + y + \frac{dy}{dx}.$$

This means:

$$(x + 1 - 4y)\frac{dy}{dx} = 2x + 2 - y, \text{ i.e.,}$$

$$\frac{dy}{dx} = \frac{2x + 2 - y}{x + 1 - 4y}$$

(3c) (2 marks) Find the equation of the tangent line at a point (a, b) on the curve (your answer should contain a and b).

Solution:

The expression for dy/dx gives the slope of the tangent line when we put $(x, y) = (a, b)$. Furthermore, the line should pass through $(x, y) = (a, b)$. We can arrange for that by setting

$$y = y'(a, b)(x - a) + b, \text{ i.e.,}$$

$$y = \frac{2a + 2 - b}{2a + 1 - 4b}(x - a) + b$$

(3d) (4 marks) Find a point on the curve *different from* $(0, 0)$ whose tangent line is parallel to the tangent line of the curve at the point $(0, 0)$.

Solution: Two lines are parallel if their slopes are equal (or if both are vertical). The slope of the tangent line at $(x, y) = (0, 0)$ is

$$y' = \frac{2 \cdot 0 + 2 - 0}{2 \cdot 0 + 1 - 4 \cdot 0} = 2.$$

In order to find all points where the tangent line has slope 2, we solve

$$2 = \frac{2x + 2 - y}{x + 1 - 4y},$$

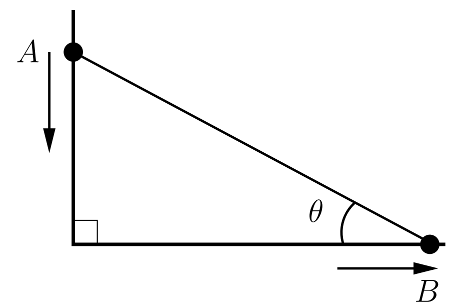
i.e., $2x+2-8y = 2x+2-y$. This means that $-8y = -y$, which amounts to $y = 0$. The only points with $y = 0$ on the curve are obtained by solving:

$$x^2 + 2x + 2\cdot 0^2 = x\cdot 0 + 0,$$

so we find $x = 0$ and $x = -2$ as solutions. Therefore, the only point *different from* $(0,0)$ where the tangent line has slope 2, is

$$(x, y) = (-2, 0)$$

4) (6 marks) Two people A and B are walking along straight lines that meet at a right angle (see diagram). A approaches the intersection at 2m/s, while B moves away from the intersection at 1m/s. At what rate is the angle θ changing when A is 10m from the intersection and B is 20m from the intersection?



Solution:

Let $y(t)$ be the position of A, and $x(t)$ be the position of B. Then $y'(t) = -2$ and $x'(t) = 1$.

Now, $\tan\theta = \frac{y}{x}$, and so

$$\frac{d}{dt}\Big(\tan\theta\Big) \;=\; \frac{d}{dt}\left(\frac{y}{x}\right)$$

$$\Longrightarrow \theta' \;=\; \frac{1}{\sec^2\theta}\frac{y'x - yx'}{x^2} \;=\; \frac{20^2}{10^2+20^2}\frac{(-2)(20)-(10)(1)}{20^2} \;=\; -\frac{1}{10} \text{ radians per second}$$

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5) (4 marks) Use the linear approximation of $f(x)$ at $x = 9$ to estimate $f(9.01)$ where

$$f(x) \;=\; \sqrt{x} + \frac{1}{\sqrt{x}}.$$

Solution:

$$a = 9, f(9) = 3\frac{1}{3},\;\; f'(9) = \frac{8}{54}$$

$$L(x) \;= f(a) + f'(a)(x-a) \;\Longrightarrow\; L(9.01) = 3\frac{1}{3} + \frac{8}{54}(0.01)$$

$$f(9.01) \approx L(9.01)$$