1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. Every correct answer will receive ½. **[3 marks]**

a) [T] If f is differentiable on R, then $f(b)-f(1)=f'(c)\cdot(b-1)$ for any number $b>1$ and some number c in the interval $(1,b)$.

b) [F] If $f'(x)>0$ on the open interval (a,b), then f is concave upward on (a,b).

c) [F] If f is continuous on the closed interval $[0,1]$ and $f(0)=f(1)$, then $f'(c)=0$ for some number c in $(0,1)$.

d) [T] If $f'(0)=0$ and $f''(0)>0$, then the function $y=f(x)$ has a local minimum value $f(0)$.

e) [F] If $f''(0)=0$ or $f''(0)$ does not exist, then the point $(0,f(0))$ is an inflection point of the function $y=f(x)$.

f) [F] The function $y=\sin x$ is one-to-one on $\mathbb{R}$.

2. State the iterative formula of Newton's Method for a function f. **[3 marks]**

initial guess x_0

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \in I_{>0}$$

3. Find $\frac{dy}{dx}$ given the following relations: **[5 marks each]**

a) $2e^x + e^y = 3e^{x-y}$, use implicit differentiation.

$$2e^x + e^y \frac{dy}{dx} = 3e^{x-y}\left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{3e^{x-y} - 2e^x}{3e^{x-y} + e^y}$$

b) $y = (\ln x)^{\ln x}$, use logarithmic differentiation. Simplify your answer.

$$\ln y = \ln x \ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\ln(\ln x)}{x} + \frac{\ln x}{x \ln x}$$

$$\frac{dy}{dx} = y\left[\frac{\ln(\ln x) + 1}{x}\right]$$

$$= (\ln x)^{\ln x}\left[\frac{\ln(\ln x) + 1}{x}\right]$$

4. Does $f(x)=|x-2|+3$ on $[1,4]$ satisfy the Mean Value Theorem? Give reasons why it does or does not. **[2 marks]**

f is not differentiable at $x = 2 \in [1,4]$.

Hence, f does not satisfy MVT on $[1,4]$.

5. The function $f(x)=x^2+1$ is defined on $[1,2]$. **[4 marks total]**

a) Show that f is one-to-one. **[1 mark]**

Assume $f(x_1)=f(x_2)$.

$x_1^2+1 = x_2^2+1$

$x_1^2 = x_2^2$

$x_1 = x_2$ on $[1,2]$ $\quad x>0$

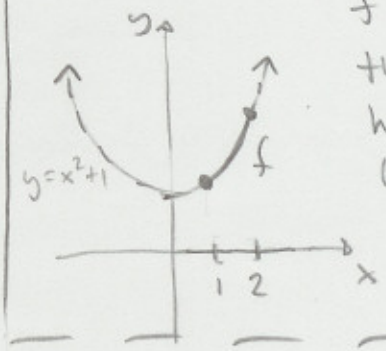
or

$f'(x) = 2x$

$2x > 0$ on $[1,2]$

$\Rightarrow$ f is increasing

or



f passes the horizontal line test.

Hence f is one-to-one.

b) Find its inverse function $g(x)$. **[3 marks]**

$y = x^2 + 1$

$x = y^2 + 1$

$x - 1 = y^2$

$\sqrt{x-1} = y$

$g(x) = \sqrt{x-1}$

6. The sum of two non-negative real numbers is 10. Find the minimum possible value of the sum of their squares. **[6 marks]**

Let x and y be the two non-negative numbers.

$x + y = 10 \Rightarrow y = 10 - x$

$S = x^2 + y^2$

$S(x) = x^2 + (10 - x)^2$

$S(x) = 2x^2 - 20x + 100 \quad , \quad x \geq 0$

$S'(x) = 4x - 20 = 0 \Rightarrow x = 5$

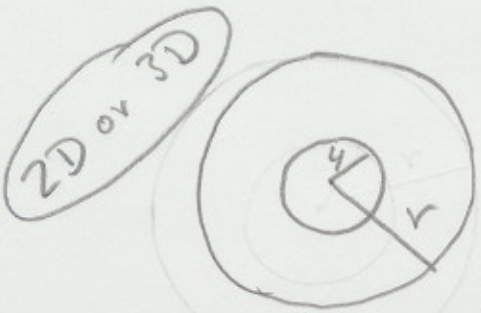
$S''(x) = 4 > 0 \Rightarrow S(5)$ is a minimum

$S(5) = 50$

The minimum possible value of the sum is 50.

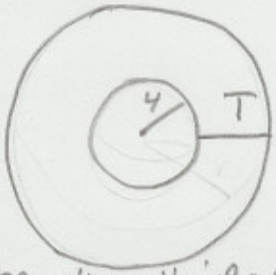
7. A spherical iron ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 10 cubic inches per minute, how fast is the thickness of the ice decreasing when it is 2 inches thick? **Do not solve**, answer the following: **[4 marks total]**

a) Draw a labeled diagram of the problem. **[1 marks]** Let statement.



Let r be the radius of the ball in inches.

or



Let T be the thickness of the ice in inches.

Let V be the volume of the ice in cubic inches.

b) Find an equation relating all pertinent variables. **[1 marks]**

$V = \frac{4}{3}\pi(r^3 - 4^3)$ or $V = \frac{4}{3}\pi((T+4)^3 - 4^3)$

c) State the known rate(s) of change. **[1 mark]**

$\frac{dV}{dt} = -10 \text{ in}^3/\text{min}$

d) Which rate of change is the problem looking for? **[1 mark]**

$\left.\frac{d(r-4)}{dt}\right|_{r=6} = \left.\frac{dr}{dt}\right|_{r=6}$ or $\left.\frac{dT}{dt}\right|_{T=2}$

8. Given $y=\frac{x^2}{x-1}$, $y'=\frac{x^2-2x}{(x-1)^2}$, and $y''=\frac{2}{(x-1)^3}$. **[2 marks each]**

a) Find the intervals of increase and decrease.

$y'=0 \Rightarrow x^2-2x=0 \Rightarrow x=0,2$

y' DNE $\Rightarrow x=1$

	0	1	2	
y'	+	−	−	+
y	incr.	decr.	decr.	incr.

y is increasing on $(-\infty,0) \cup (2,\infty)$

y is decreasing on $(0,1) \cup (1,2)$

b) Find the intervals of concave up and concave down.

$y''=0 \Rightarrow$ no such x

y'' DNE $\Rightarrow x=1$

	1	
y''	−	+
y	c.d.	c.u.

y is concave up on $(1,\infty)$

y is concave down on $(-\infty,1)$

c) Find the inflection point(s), if any.

y is not defined at $x=1$

No inflection points.

d) Circle which graph below best describes the above curve.