Solutions

Math 151-D1 Calculus I
Midterm 1, February 8
Instructor: Matt DeVos

Full Name (please print): _____

Student Number: _____

Signature: _____

1. DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO
2. Fill in the above box.
3. This exam contains 7 pages with 6 questions. Once the exam begins, check that your exam is complete.
4. No book, paper, or device other than the usual writing instruments, this booklet, and scientific calculators are allowed. **No graphing or programmable calculators are permitted.**
5. During this examination, speaking to, communicating with, or exposing written papers to the view of other students is forbidden.
6. You may use the back of the previous page for rough work or if you run out of space.
7. Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.

Problem	Score	Value
1		4
2		6
3		3
4		4
5		6
6		5
Total:		28

Problem 1. *(4 points)* Mark each statement as true (**T**) or false (**F**)

T If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

F The function $\sin x$ is one-to-one.

F If $\lim_{x\to\infty} f(x) = L$, then the graph of $f(x)$ has a vertical asymptote at $x = L$.

T Every polynomial which has both positive and negative values has at least one real root.

Problem 2. *(6 points)* Evaluate the following limits.

(2 points) $\lim_{x\to 2^+} \frac{5-x^2}{x-2}$.

$$\lim_{x\to 2^+} \frac{5-x^2 \to 1}{x-2 \to 0^+} = \infty$$

(2 points) $\lim_{x\to 2} \frac{\sqrt{x^3+1}}{x+1}$.

$$\lim_{x\to 2} \frac{\sqrt{x^3+1} \to 3}{x+1 \to 3} = 1$$

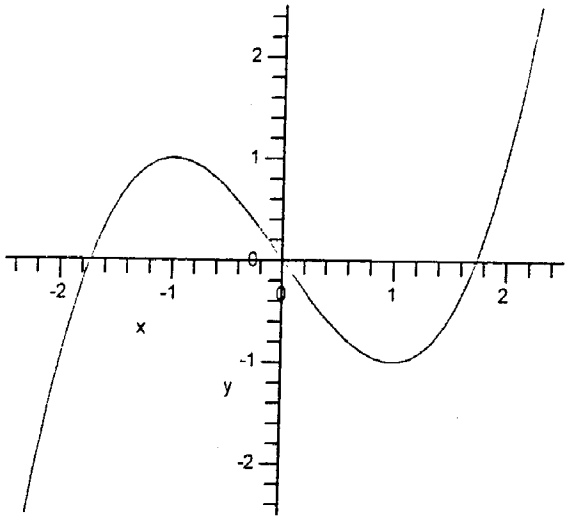
(2 points) $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$.

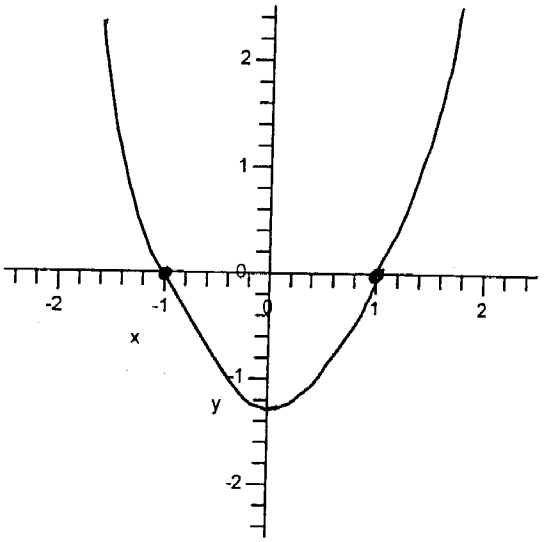
$$= \lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \left(\frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} \right)$$

$$= \lim_{x\to 6} \frac{(x-2)-4}{(x-6)(\sqrt{x-2}+2)}$$

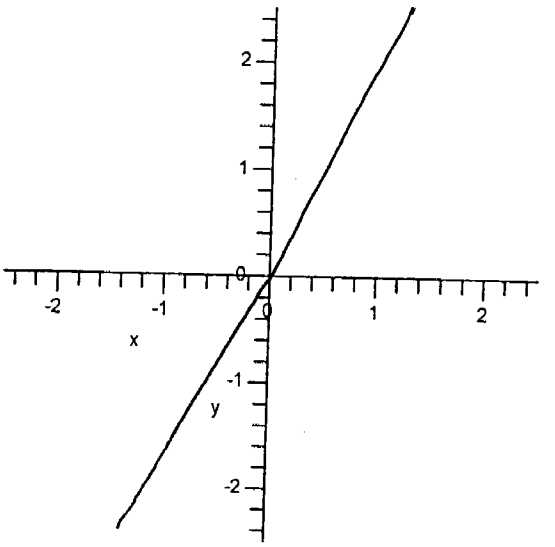
$$= \lim_{x\to 6} \frac{1}{\sqrt{x-2}+2} = \frac{1}{4}$$

Problem 3. *(3 points)* Let $f(x)$ be the function given in the diagram below. Give a rough sketch of the functions $f'(x)$ and $f''(x)$.





$f'(x)$



$f''(x)$

Problem 4. *(4 points)* Find $\frac{dy}{dx}$ for the following functions.

(2 points) $y = \frac{x^3+1}{\tan x}$

$$\frac{dy}{dx} = \frac{\tan x\,(x^3+1)' - (\tan x)'(x^3+1)}{\tan^2 x}$$

$$= \frac{\tan x\,(3x^2) - \sec^2 x\,(x^3+1)}{\tan^2 x}$$

(2 points) $y = e^{x^2+\cos x}$

$$\frac{dy}{dx} = e^{x^2+\cos x}\,(x^2+\cos x)'$$

$$= e^{x^2+\cos x}\,(2x - \sin x)$$

Problem 5. *(6 points)* Let $f(t) = t^3 - 6t^2 + 2$ be the position of a particle at time t.

(2 points) What is the average velocity over the interval $[1, 2]$?

avg. vel over $[1,2]$ is $\dfrac{f(2) - f(1)}{2 - 1}$

$= \dfrac{(8 - 24 + 2) - (1 - 6 + 2)}{1} = -11$

(2 points) What is the instantaneous velocity at $t = 2$? (this may be computed using any technique you have learned in class).

inst. vel at 2 is $f'(2)$

$f'(t) = 3t^2 - 12t$

So $f'(2) = 12 - 24 = -12$

(2 points) Where is the velocity of the particle (strictly) increasing?

velocity is str. incr. when $f''(t) > 0$

$f''(t) = 6t - 12$

$= 6(t - 2)$

positive when $t > 2$

or equiv. when t is in $(2, \infty)$

Problem 6. *(5 points)* Let $f(x) = c(x-2)^2$.

(3 points) Find the equation of the line tangent to $f(x)$ at $x = 3$.

the slope of tang. is $f'(3)$

$f'(x) = c2(x-2)$

so $f'(3) = 2c$

the tang. line passes through $(3, f(3)) = (3, c)$

so it is given by $y - c = 2c(x-3)$

or $y = 2cx - 5c$

(2 points) For what value(s) of c does the line tangent to $f(x)$ at $x = 3$ pass through the point $(4, 9)$?

the tang. line passes through $(4,9)$

when $9 = 2c(4) - 5c$

$9 = 3c$

$\boxed{c = 3}$