Simon Fraser University

Math 151 Section D1
Fall 2006
October 4, 2006

Instructor: V. Jungic Date: October 4, 2006

Last Name: _____

First Name: _____

E-mail: _____

Signature: _____

Instructions

1. Fill in the information above.
2. Please do not open the examination booklet until you are told to do so.
3. Do all your work in this test booklet. Show all your work.
4. Please no books, no notes, and no calculators.

1	2	3	4	5	6	Total
						/30

1. **[3 marks]** Mark each statement **T** (True) or **F** (False):

T **F** If $f(x) < 0$ for all x and $\lim_{x\to 0} f(x)$ exists then $\lim_{x\to 0} f(x) < 0$.

T **F** A function f is differentiable at c if its graph has a tangent line at the point $(c, f(c))$.

T **F** If $\lim_{x\to 1} f(x) = \infty$ and $\lim_{x\to 1} g(x) = \infty$ then $\lim_{x\to 1}[f(x)/g(x)]$ must be equal to 1.

T **F** A function can have three different horizontal asymptotes.

T **F** A function f is differentiable on an open interval (a, b) if it is differentiable at every number in that interval.

T **F** If f is differentiable at c, then f is continuous at c.

2. (a) **[1]** Show by means of an example that $\lim_{x\to 1}(f(x)\cdot g(x))$ may exist even though neither $\lim_{x\to 1} f(x)$ nor $\lim_{x\to 1} g(x)$ exists. Justify your answer.

(b) **[1]** Draw a graph of a function with a removable discontinuity.

(c) **[1]** Give an example of a function f that is continuous for all real numbers and such that f' is not defined at $x = 3$. Write a formula and draw a graph of f.

3. Evaluate the following limits. **Justify your answers**.

(a) **[2]** $\lim_{x\to\pi}(x-\pi)\cos\frac{1}{\pi-x}$

(b) **[2]** $\lim_{x\to 5}\frac{25-x^2}{2x^2-17x+35}$

(c) **[2]** $\lim_{x\to\infty}\frac{3x^3-2x^2+2x-1}{4-2x+5x^2-2x^3}$

4. (a) **[2]** State the Intermediate Value Theorem.

(b) **[4]** Show that the equation $x^3 - 4x + 1 = 0$ has three different roots by calculating the values of the left-hand side at $x = -3, -2, -1, 0, 1, 2, 3$ and then applying the Intermediate Value Theorem.

5. (a) **[2]** Define the derivative of a function f at a number a.

(b) **[4]** Let $f(x) = \sqrt{2-x}$. Use the definition of the derivative to find $f'(-2)$.

(c) **[2]** Find an equation of the tangent line to the curve $y = f(x) = \sqrt{2-x}$ at the point $(-2, f(-2))$.

6. Let $f(3) = 2$, $g(3) = 2$, $f'(3) = -1$ and $g'(3) = -2$. Evaluate:

   (a) **[2]** $F'(3)$ if $F(x) = e^x g(x) - \sqrt[3]{x^2} f(x)$.

   (b) **[2]** $G'(3)$ if $G(x) = \frac{f(x)+x^3}{1-g(x)}$.