1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. Every correct answer will receive ½ and every incorrect answer will receive -½. **[3 marks]**

a) If $\lim_{x \to a} f(x) = f(a)$ then f is differentiable at $x = a$. F

b) As $x \to 0$ the limit of $\sin\frac{\pi}{x}$ does not exit. T

c) A polynomial function is continuous for all real numbers. T

d) The function $f(x) = |x|$ is differentiable for all real numbers. F

e) If $f'(x) > 0$ then f is decreasing. F

f) If f has a horizontal tangent at $x = a$ then $f'(a) = 0$. T

2. State the Squeeze Theorem. **[3 marks]**

Suppose that $f(x) \leq g(x) \leq h(x)$ ~~for al~~

for all $x \neq a$ around a

and that $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$,

then $\lim_{x \to a} g(x) = L$.

3.

4. Find the following limits if they exist. Show your work.
[3 marks each = 9 marks]

a) $\lim_{x \to -3^-} \frac{x+3}{|x+3|}$

$$|x+3| = \begin{cases} x+3, & x \geq -3 \\ -(x+3), & x < -3 \end{cases}$$

$$= \lim_{x \to -3^-} \frac{x+3}{-(x+3)} = \lim_{x \to -3^-} (-1) = -1$$

b) $\lim_{x \to 3^+} \frac{9-x^2}{x-3}$

$$= \lim_{x \to 3^+} \frac{(3-x)(3+x)}{x-3} = \lim_{x \to 3^+} -(3+x) = -6$$

c) $\lim_{x \to 0} \frac{1+x^2-\cos^2 x}{x^2}$ $= \lim_{x \to 0} \left(\frac{x^2}{x^2} + \frac{1-\cos^2 x}{x^2} \right)$

$$= \lim_{x \to 0} \left(1 + \frac{\sin^2 x}{x^2} \right)$$

$$= 1 + \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^2$$

$$= 1 + 1^2 = 2$$

5. Let $f(x)=\begin{cases} ax^2, if\ x \geq 2 \\ 3x+a, if\ x<2 \end{cases}$.

a) For what value(s) of a is f continuous at $x=2$? Show your work. **[3 marks]**

reasoning with words

The limit as $x \to 2$ must exist (left side limit and right side limit must exist and be the same) and equal the function value at $x=2$ for f to be cont. at $x=2$.

reasoning with math

$\lim_{x\to 2^+} f(x) = \lim_{x \to 2^+} ax^2 = 4a$

$\lim_{x\to 2^-} f(x) = \lim_{x \to 2^-} (3x+a) = 6+a$

$f(2) = 4a$

$4a = 6+a \Rightarrow a = 2$

f is only continuous for $a=2$ at $x=2$.

b) Is f differentiable at $x=2$? Provide a reason. **[1 mark]**

f is not differentiable at $x=2$ because

... there is a cusp for $a=2$ (jump disc. for $a \neq 2$)

or ...

6. Use the **definition of the derivative** for the following question.

a) State the definition of the derivative. **[1 mark]**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x+3}$. **[3 marks]**

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$$

$$= \lim_{h \to 0} \frac{x+h+3-(x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}}$$

$$= \frac{1}{2\sqrt{x+3}}$$

7. Find $\frac{dy}{dx}$ if **[3 marks each = 9 marks]**

a) $y = \sqrt{(x-3)^2}$

$$y' = \frac{1}{2\sqrt{(x-3)^2}} \cdot 2(x-3) = \frac{x-3}{\sqrt{(x-3)^2}}, \quad x \neq 3$$

y' DNE at $x = 3$.

b) $y = x\sec(3x)$

$$y' = 1 \cdot \sec(3x) + 3x\tan(3x)\sec(3x)$$

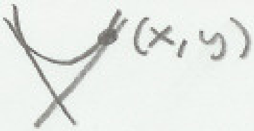
c) $y = \dfrac{e^{3x}}{\cos x}$

$$y' = \frac{3e^{3x}\cos x + e^{3x}\sin x}{\cos^2 x}$$

$$\left[= e^{3x}(3\sec x + \sec x \tan x)\right]$$

of form $y = mx + b$?

8. Find the tangent line(s) to the function $y = x^2 + 4$ that pass through the point $(0,0)$. **[4 marks]**



slope: $y' = 2x$

formula: $\frac{y-0}{x-0} = 2x$ OR $(y-2) = 2x(x-0)$ $\Rightarrow y = 2x^2$

find x: $2x^2 = x^2 + 4$ $\Rightarrow$ $x = \pm 2$

tangent line equations:

$y = 4x$ and $y = -4x$

9. Find the maximum and minimum values of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the interval $[0,2]$. **[4 marks]**

$f'(x) = 6x^2 + 6x - 12$

critical numbers: $6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

reject, not in interval

end points: $x = 0, 2$

$f(0) = 4$

$f(1) = -3$ ← minimum value

$f(2) = 8$ ← maximum value