Simon Fraser University Department of Mathematics Burnaby Campus

MATH 151, Summer 2006 **Final** August 9th, 2006, **15:30 – 18:30**

Last Name (please print): _____

First Name (please print): _____

Student ID: _____

Email ID: _____@sfu.ca

Instructor: Keshav Mukunda

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 17 pages with a total of 10 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK! No credit will be given for just writing down the answer.
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Do not write in this table!	
Question	**Marks**
1	**/12**
2	**/13**
3	**/6**
4	**/17**
5	**/10**
6	**/8**
7	**/6**
8	**/10**
9	**/8**
10	**/10**
Total	**/100**

Q1.
Evaluate the following.

(a) [3 marks] $\lim_{x \to -\infty} \dfrac{3x^6 - 7x^5 + x}{5x^6 + 4x^5 - 3}$

(b) [3 marks] $\lim_{x \to 4} f(x)$, where $x + 2 \leq f(x) \leq x^2 - 10$ for all values of x.

Q1. (continued)

(c) [3 marks] $\lim_{x \to 0^+} (\sin x)(\ln \sin x)$

(d) [3 marks] $\lim_{x \to 0} (\cosh x)^{1/x^2}$

Q2.
The following questions concern derivatives.

(a) [3 marks] Find dy/dx , if $\sinh x - \cos y = x^2 y$

Your answer may be left in terms of both x and y. Do not simplify.

(b) [4 marks] Find f , if $f'(x) = 2\cos x + 8x^3 - e^x$ and $f(0) = 7$.

Q2. (continued)

(c) [6 marks] Find y' , when $y = \dfrac{(x+2)^{3\ln x}}{(x^2+1)^{1/2}}$

Your final answer **must** be expressed only in terms of x, but **don't** try to simplify it.

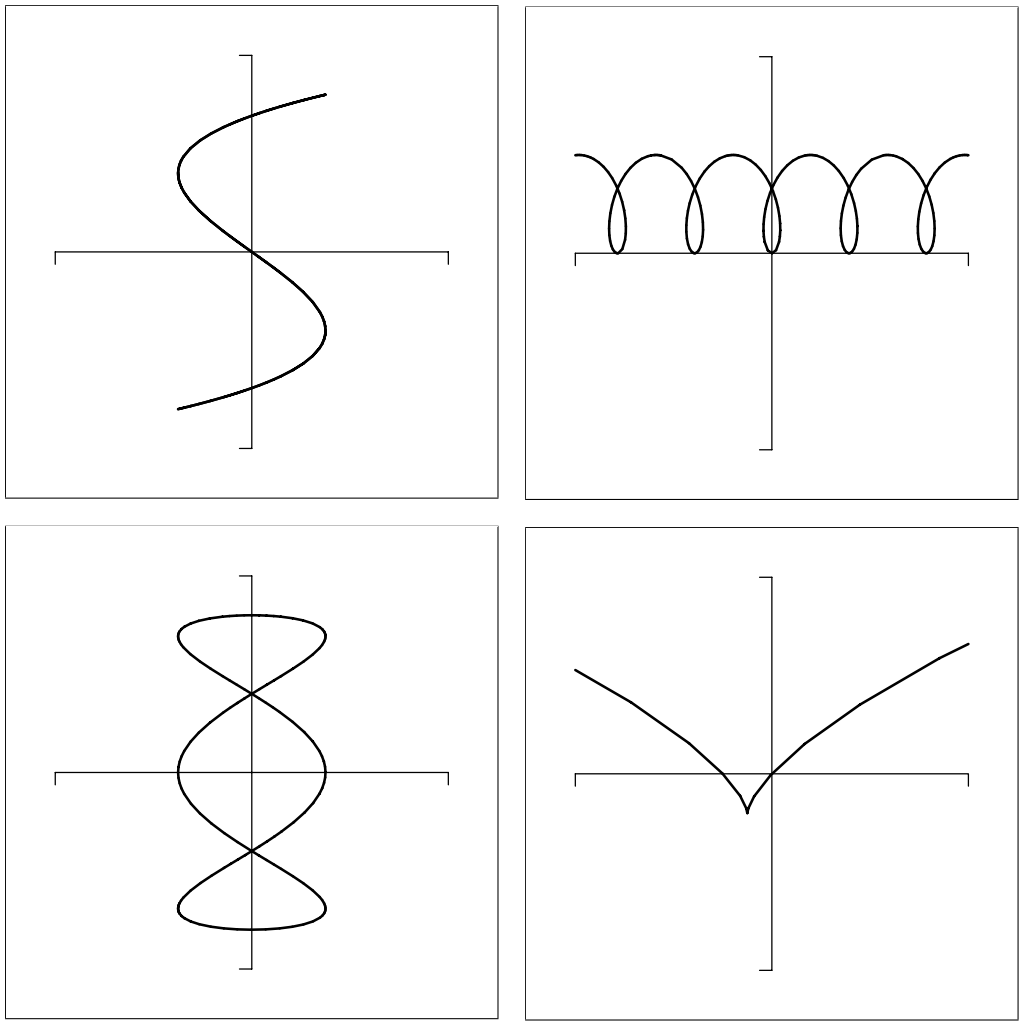
Q3.
[6 marks] Four parametric curves are shown below. Two of them have parametric equations given by

(A) $x = 3\cos 6t$ $\qquad$ $y = 4\sin 2t$

(B) $x = t - \sin 2t$ $\qquad$ $y = 1 - \cos 2t$

Identify the curves that correspond to **(A)** and **(B)**. (No explanation required.)

(A) _____ **(B)** _____



Q4.

In this question we consider the function $f(x) = \dfrac{x-3}{\sqrt{x^2-9}}$.

You can also make use of the following derivatives:

$$f'(x) = \frac{3x-9}{(x^2-9)^{3/2}} \qquad \text{and} \qquad f''(x) = \frac{-6x^2+27x-27}{(x^2-9)^{5/2}}$$

Answer parts (a)-(i) to construct the graph of $y = f(x)$.

(a) [1 mark] Find the domain of $f(x)$.

(b) [1 mark] Find the coordinates of all x- and y-intercepts, if any.

Q4. (continued)

(c) [2 marks] Find all horizontal and vertical asymptotes, if any.

(d) [1 mark] Find all critical numbers, if any.

Q4. (continued)

(e) [3 marks] Find the intervals on which f is increasing and those on which f is decreasing.

(f) [1 mark] Find the (x, y) coordinates of all maximum and minimum points, if any.

Q4. (continued)

(g) [3 marks] Find the intervals on which f is concave up and those on which f is concave down.

(h) [1 mark] Find the (x, y) coordinates of all inflection points, if any.

Q4. (continued)

(i) [4 marks] Finally, sketch the graph of $y = f(x)$ using all of the above information. All relevant points and lines **must be labelled**.

Q5.
[10 marks] Verify that the function

$$g(x) = \frac{3x}{x+7}$$

satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem. Leave your final answer(s) exact, do not use a calculator to evaluate them.

Q6.

(a) [4 marks] Find the linear approximation to the function $f(x) = \sqrt{(x+4)^3}$ at $a = 0$.

(b) [4 marks] Use this approximation to estimate the number $\sqrt{(3.95)^3}$. Is your estimate an overestimate or an underestimate?
(Hint: What is the concavity of the function $f(x)$?)

Q7.

(a) [2 marks] State Newton's iterative formula that produces a sequence of approximations $x_1, x_2, x_3, \ldots$ to a root of a function $f(x)$.

(b) [4 marks] Find the positive root of the equation $\cos x = x^2$ using Newton's method, correct to 3 decimal places, with the first approximation $x_1 = 1$.

Q8.

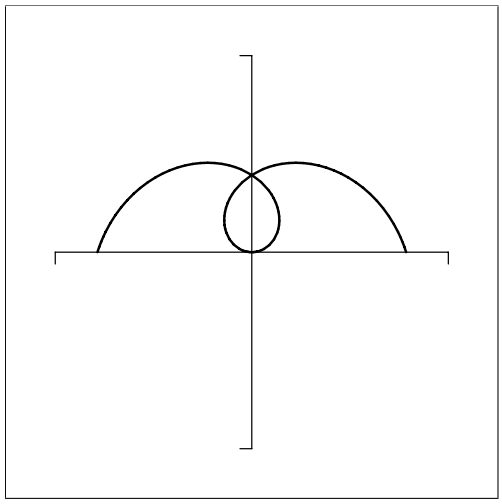
[10 marks] A straight piece of wire 40 cm long is cut into two pieces. One piece is bent into a circle and the other is bent into a square. How should the wire be cut so that the total area of both circle and square is **minimized**?

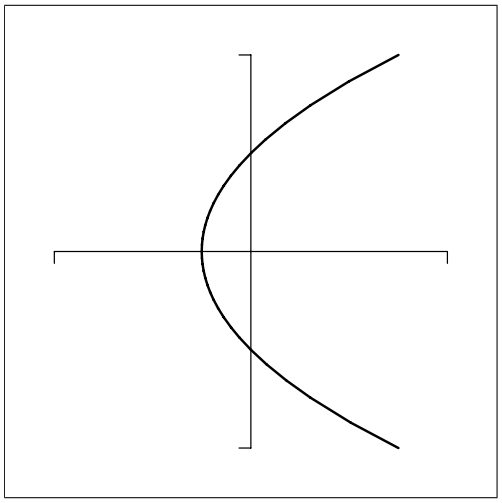
Q9.
[8 marks] Match each polar curve to its corresponding equation. Each curve is plotted for $-\pi \leq \theta \leq \pi$. (No explanation is required.)

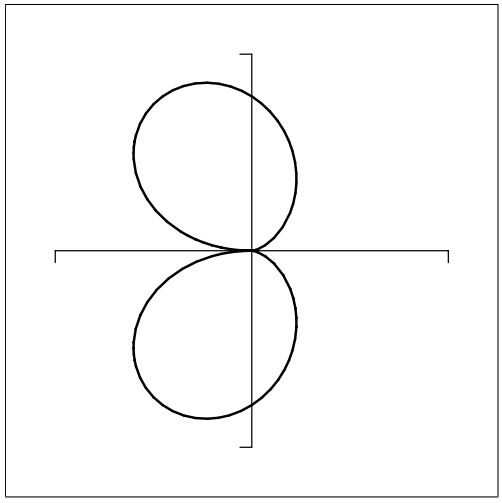
(A) $r = \theta \sin\theta$ **(B)** $r = \dfrac{1}{1+\frac{1}{2}\cos\theta}$

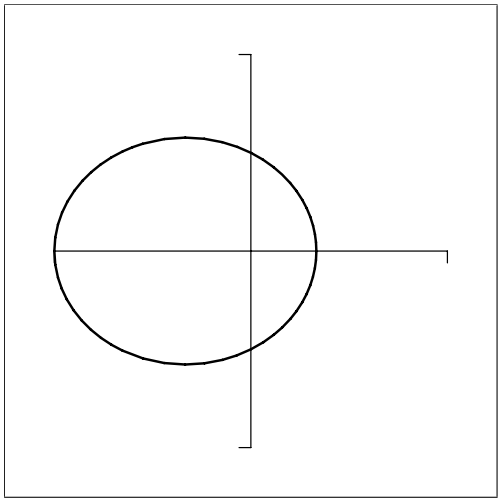
(C) $r = \dfrac{\theta}{2}$ **(D)** $r = \dfrac{1}{1-\cos\theta}$

(A) _____ **(B)** _____ **(C)** _____ **(D)** _____









Q10.
[10 marks] A ladder 15 ft long rests against a vertical wall. Its top slides down the wall while its bottom moves away along the level ground at a speed of 2 ft/s. How fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/3$ radians?
