SIMON FRASER UNIVERSITY
DEPARTMENT OF MATHEMATICS

Math 151 – Calculus I

Instructor: Bruce Kadonoff

FINAL EXAM

5 August 2005

Name: ________________________

Student Number: _______________

Question	Marks
1	/4
2	/4
3	/4
4	/6
5	/6
6	/8
7	/6
8	/6
9	/6
10	/8
11	/12
12	/8
13	/8
14	/8
15	/6
TOTAL	/100

Instructions:

1. Do not open this test booklet until instructed to do so.
2. You have three (3) hours for this exam.
   There is a total of 100 marks, ensure you allocate your time effectively.
3. You are allowed to use a basic scientific calculator (that is, no graphing or programming functions).
4. Answers may be written in pen or pencil. Errors or changed answers must be clearly erased or crossed out.
5. Only work shown on this test paper will be marked.
6. You are not allowed any reference material including dictionaries. If you are caught with reference material, you will receive a zero on the exam and may face additional disciplinary action by the school administration.
7. Copying from or communicating with a neighbour will result in both students receiving a zero and may result in further disciplinary action by the school administration.
8. If you finish early, hand in your test and leave the room immediately.

1. [4 marks] Find constants a and b such that $\lim_{x\to 0}\frac{\sqrt{ax+b}-2}{x}=1$.

2. [4 marks] Use the squeeze theorem to show that $\lim_{x \to 0^+} \left(\sqrt{x}\, e^{\sin(1/x)} \right) = 0$.

3. [4 marks] Prove that $f(x) = \dfrac{\ln x}{x}$ has horizontal asymptote $y = 0$.

4. [6 marks] The function $f(x) = \begin{cases} e^x & x \le 1 \\ mx + b & x > 1 \end{cases}$ is continuous and differentiable at $x = 1$. Find values for the constants m and b.

5. [6 marks] At what point on the curve $y = \sinh(x)$ does the tangent line have a slope of 1?

6. [8 marks] Find the point on the curve $x + y^2 = 0$ that is closest to the point (0 , -3).

7. [6 marks] For what values of the constant c does $\ln x = cx^2$ have solutions? Assume that $c > 0$.
Hint: begin with a sketch and find where it has one solution

8. [6 marks] A lighthouse is located on a small island three (3) km off-shore from the nearest point **P** on a straight shoreline. Its light makes four (4) revolutions per minute. How fast is the light beam moving along the shoreline when it is shining on a point one (1) km along the shoreline from **P**?

9. [6 marks] The tangent line to the graph $y = f(x)$ at the point $A(2, -1)$ is given by $y = -1 + 4(x - 2)$. It is also known that $f''(2) = 3$.

a) Assume that Newton's Method is used to solve the equation $f(x) = 0$ and that $x_0 = 2$ is the initial guess. Find the next approximation, x_1, to the solution.

b) Assume that Newton's Method is used to find a critical point for f and that $x_0 = 2$ is the initial guess. Use Newton's Method to find the next approximation, x_1, to the critical point.

10. [8 marks] The equation $y(x-3)+e^{y}=x^{2}-15$ defines *y* implicitly as a function of *x* near the point A(4 , 0).

a) Determine the values of *y'* and *y''* at this point.

b) Use a tangent line approximation to estimate *y* when *x* = 3.95.

c) Is the true value of *y* greater or less than the approximation in part (b)? Make a sketch showing how the curve relates to the tangent line near the point A(4 , 0).

11. [12 marks] Let $y = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$.

a) Find the domain and intercepts.

b) Find all the asymptotes.

c) Find the intervals where the function is increasing or decreasing.

d) Find the intervals where the function is concave up or concave down.

e) Sketch the graph of the function.

12. [8 marks] The rate at which a student learns new material is proportional to the difference between a maximum, **M**, and the amount he already knows at time t, **A**(t). This is called a learning curve.

a) Write a differential equation to model the learning curve described.

b) Solve the differential equation you created in part (a).

c) If it took a student 100 hours to learn 50% of the material in Math 151 and he would like to know 75% in order to get a B, how much longer should he study? You may assume that the student began knowing none of the material and that the maximum he might achieve is 100%.

13. [8 marks] Given the parametrically defined curve: $\begin{cases} x = t(t^2 - 3) \\ y = 3(t^2 - 3) \end{cases}$,

a) Find the y-intercepts of the curve.

b) Find the points on the curve where the tangent line is horizontal or vertical.

b) Sketch the curve.

14. [8 marks] Find the slope of the line tangent to the polar curve r = 2 at the points where it intersects the polar curve $r = 4\cos\theta$. Hint: After you find the intersection points, convert one of the curves to a pair of parametric equations with θ as the parameter.

15. [6 marks] If a and b are positive numbers, find the x-coordinate which gives the absolute maximum value of $f(x) = x^a(1-x)^b$ on the interval $0 \le x \le 1$.