Simon Fraser University

Math 151-3, Summer 04
Final Exam

Date: 4 Aug. 2004
Time: 15:30 - 18:30
Place: WMC 3260

Last Name_____ Given Names_____

Student Number_____

Instructions

1. Do not open this test booklet until instructed to do so.
2. Print your name and write your student number above.
3. No calculators or other calculating devices may be used.
4. Full marks will be awarded for correct, complete and well-organized solutions.
5. You may use the back of any page for rough work.
6. There are 13 pages in this test booklet.

Question	1	2	3	4	5
Marks	/8	/8	/10	/11	/9

Question	6	7	8	9	10
Marks	/10	/9	/11	/15	/9

Total_____/100

Good Luck!

1 a) Let $f(x)=\begin{cases}\dfrac{x^2-1}{|x-1|}, & x\neq 1\\ 4, & x=1\end{cases}$, find $\lim\limits_{x\to 1^-} f(x)$ (4 marks)

b) Find $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$. (4 marks)

2 a) Find the domain of the function $f(x) = \dfrac{\ln[\ln(\ln x)]}{x-3} + \sin x$. (4 marks)

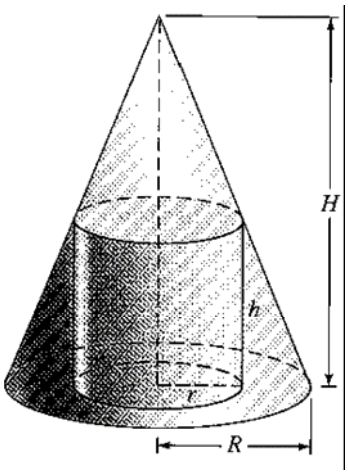
b) Given $F(x) = f^2[g(x)]$, $g(1) = 2$, $g'(1) = 3$, $f(2) = 4$, and $f'(2) = 5$, find $F'(1)$. (4 marks)

3 a) Given $y = \tan^{-1}(\cosh\sqrt{x^2+1})$, find y'. (5 marks)

b) A boy starts walking north at a speed of 1.5 m/s, and a girl starts walking west at the same point P at the same time at a speed of 2 m/s. At what rate is the distance between the boy and the girl increasing 6 seconds later? (5 marks)

4 a) Use a linear approximation to estimate $\ln 0.9$. (5 marks)

b) Find the dimensions of the right circular cylinder of maximum volume that can be inscribed in a right circular cone of radius R and height H. (6 marks)



5 a) Prove that $f(x) = \frac{1}{(x+1)^2} - 2x + \sin x$ has exactly one positive root.

(5 marks)

b) Use the mean value theorem to show that
$|\sin b - \sin a| \leq |b - a|$, for all real numbers a and b. (4 marks)

6 Given the parametric curve

$x = e^t$, $y = e^{-t}$.

a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (6 marks)

b) Find the equation of the line tangent to the curve that is parallel to the line $y + x = 1$. (4 marks)

7 a) Express the polar equation $r = \cos 2\theta$ in rectangular coordinates.

(4 marks)

b) Sketch the graph of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$, and determine its foci.

(5 marks)

8 a) Find the solution of the initial value problem

$$\frac{dy}{dx} = \sqrt{1 - y^2}\,,\ y(0) = 1.$$ (6 marks)

b) In a certain culture of bacteria, the number of bacteria increased tenfold in 10 h. Assuming natural growth, how long did it take for their number to double? (5 marks)

9 Given $f(x) = \dfrac{x^2 - 1}{x}$,

a) Find the domain and $x-$intercepts. (2 marks)

b) Find all asymptotes. (3 marks)

c) Determine the intervals on which the function is increasing or decreasing. Find the local maximum and minimum, if they exist. (4 marks)

d) Determine the intervals on which the function is concave upward or downward. Find the inflection points, if they exist. (3 marks)

e) Sketch the graph. (3 marks)

10 a) If $g(x)$ is continuous (but not differentiable) at $x = 0$, $g(0) = 8$, and $f(x) = x \cdot g(x)$, find $f'(0)$. (4 marks)

b) Let l be any tangent to the curve: $\sqrt{x} + \sqrt{y} = \sqrt{k}$ $(k > 0$, a constant$)$. Show that the sum of $x-$ intercept and $y-$intercept of l is k . (5 marks)

Inverse Trigonometric Functions

Function	Domain	Range
$\sin^{-1} x$	$-1 \le x \le 1$	$-\pi/2 \le y \le \pi/2$
$\cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$\tan^{-1} x$	$-\infty < x < +\infty$	$-\pi/2 < y < \pi/2$
$\cot^{-1} x$	$-\infty < x < +\infty$	$0 < y < \pi$
$\sec^{-1} x$	$\lvert x \rvert \ge 1$	$0 \le y < \pi/2,$ $\pi/2 < y \le \pi$
$\csc^{-1} x$	$\lvert x \rvert \ge 1$	$-\pi/2 \le y < 0,$ $0 < y \le \pi/2$

Some Derivative Formulas

$$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}, \ |x|>1, \qquad (\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}, \ |x|>1,$$

$$(\sinh x)' = \cosh x, \qquad (\cosh x)' = \sinh x,$$

$$(\tanh x)' = \operatorname{sech}^2 x, \qquad (\coth x)' = -\operatorname{csch}^2 x,$$

$$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x, \qquad (\operatorname{csch} x)' = -\operatorname{csch} x \coth x.$$

Hyperbolic Functions Identities

$$\cosh^2 x - \sinh^2 x = 1,$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x, \qquad \coth^2 x - 1 = \operatorname{csch}^2 x,$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y,$$

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y,$$

$$\sinh 2x = 2\sinh x \cosh x, \qquad \cosh 2x = \cosh^2 x + \sinh^2 x,$$

$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1), \qquad \sinh^2 x = \frac{1}{2}(\cosh 2x - 1).$$