Simon Fraser University
Math 151-3, Spring 2004
Final Examination (Blue version)

Time: 3 hours **April 14**

Last Name Given Names

Student Number Signature

Instructions

- Do not open this exam booklet until instructed to do so.
- Write your name, student number and signature above.
- The possession of any calculators in this exam is considered as **academic dishonesty**.
- Full marks will be awarded for correct, complete and well-organized solutions.
- You may use the back of any page for rough work.
- There are 13 pages in this test booklet.

Question	Marks
1	/8
2	/6
3	/8
4	/8
5	/10

Question	Marks
6	/16
7	/8
8	/8
9	/10
10	/8
Total	/90

1. Evaluate the following limits if they exist.

   (a) [4 marks] $\displaystyle\lim_{x\to-\infty} \frac{2x+3x^3}{x^3+2x-1}$

   (b) [4 marks] $\displaystyle\lim_{x\to 0^+} (1+7x)^{1/(5x)}$

2. Suppose the functions $F(x)$ and $G(x)$ satisfy the following properties:

$$\begin{array}{lll} F(3) = 2, & G(3) = 4, & G(0) = 3, \\ F'(3) = -1, & G'(3) = 0, & G'(0) = 2. \end{array}$$

(a) [2 marks] If $S(x) = \dfrac{F(x)}{G(x)}$, find $S'(3)$. Simplify your answer.

(b) [2 marks] If $T(x) = F[G(x)]$, find $T'(0)$. Simplify your answer.

(c) [2 marks] If $U(x) = \ln[F(x)]$, find $U'(3)$. Simplify your answer.

3. (a) [4 marks] Find $\dfrac{dy}{dx}$ if $y = \sec[\text{sech}\,(x)]$.

(b) [4 marks] Find $\dfrac{dy}{dx}$ if $\ln(x+y) = xy - y^3$. Express $\dfrac{dy}{dx}$ as a function of x and y.

4. (a) [4 marks] Find the equation of the line that is tangent to the graph of $y = \sqrt{x} - \dfrac{1}{\sqrt{x}}$ at $x = 1$.

(b) [4 marks] Use linear approximation to estimate the value of $\sqrt[3]{26^2}$. Express your answer as a single fraction (for example, 16/729).

5. An open-top box is to have a square base and a volume of 10 m^3. The cost per square meter of material is \$5 for the bottom and \$2 for the four sides. Let x and y be the lengths of the box's width and height respectively. Let C be the total cost of material required to make the box.

   (a) [4 marks] Express C as a function of x and find its domain.

   (b) [6 marks] Find the dimensions of the box so that the cost of materials is minimized. What is this minimum cost? Justify your answer.

6. For this question, consider the function

$$f(x) = \frac{x}{x^2 - 1}.$$

For your convenience, the first and second derivatives of $f(x)$ are

$$f'(x) = \frac{-x^2 - 1}{\left(x^2 - 1\right)^2} \quad \text{and} \quad f''(x) = \frac{2x\left(x^2 + 3\right)}{\left(x^2 - 1\right)^3}$$

respectively.

(a) [4 marks] Determine any horizontal and vertical asymptotes of f.

(b) [4 marks] Determine the open intervals on which f is increasing as well as those on which f is decreasing.

(c) [4 marks] Determine the open intervals on which f is concave upward as well as those on which f is concave downward.

(d) [4 marks] Based on the information found in Parts (a) to (c), sketch the graph of f. Indicate any relative extremum, inflection points and intercepts on your graph.

7. (a) [4 marks] Find the function $y\,(x)$ that is the solution of the following initial value problem:

$$\frac{dy}{dx} = y^2 + 1 \quad \text{and} \quad y\,(\pi/4) = 0.$$

(b) [4 marks] On a certain day, a scientist had 1 kg of a radioactive substance X at 1:00 pm. After six hours, only 27 g of the substance remained. How much substance X was there at 3:00 pm that same day?

8. Given the parametric curve

$$x = \cos^3 t, \qquad y = \sin^3 t.$$

(a) [4 marks] **Without** eliminating the parameter t, show that $\dfrac{dy}{dx} = -\tan t$.

(b) [4 marks] Determine the concavity of this curve when $t = 1$.

9. For this question, let $\mathcal{C}$ be the conic section described by the equation

$$x^2 - 2y^2 + 4x = 0.$$

(a) [4 marks] Using the method of completing the square, identify the conic section $\mathcal{C}$.

(b) [6 marks] Sketch the graph of $\mathcal{C}$. Indicate its center, foci and asymptotes (if any).

10. (a) [4 marks] Use the **definition of limits** to prove that

$$\lim_{x \to 0} x^3 = 0.$$

Your proof should involve the variables δ and ϵ.

(b) [4 marks] Let f be a function that is continuous everywhere and let

$$F(x) = \begin{cases} \dfrac{f(x)\sin^2 x}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Use the **definition of derivatives** to evaluate $F'(0)$. Your answer should be in terms of f. Justify your answer.

Some Derivative Formulas

$\dfrac{d}{dx}\coth x = -\operatorname{csch}^2 x$	$\dfrac{d}{dx}\cot^{-1} x = -\dfrac{1}{1+x^2}$
$\dfrac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$	$\dfrac{d}{dx}\sec^{-1} x = \dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\dfrac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$	$\dfrac{d}{dx}\csc^{-1} x = -\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$

Hyperbolic Functions Identities

$\cosh^2 x - \sinh^2 x = 1$	$\sinh(2x) = 2\sinh x \cosh x$
$\tanh^2 x + \operatorname{sech}^2 x = 1$	$\cosh(2x) = \cosh^2 x + \sinh^2 x$
$\coth^2 x - 1 = \operatorname{csch}^2 x$	$\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
	$\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$

Some Conic Sections

	Ellipse	Hyperbola
Eccentricity	$e = \dfrac{c}{a} < 1$	$e = \dfrac{c}{a} > 1$
a, b, c relation	$a^2 = b^2 + c^2$	$c^2 = a^2 + b^2$
Equation	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
Vertices	$(\pm a, 0)$	$(\pm a, 0)$
y-intercepts	$(0, \pm b)$	None
Directrices	$x = \pm\dfrac{a}{e}$	$x = \pm\dfrac{a}{e}$
Asymptotes	None	$y = \pm\dfrac{bx}{a}$