Simon Fraser University

Math 151 Section D1

Fall 2006

Final Exam

Instructor: V. Jungic

Date: December 9, 2006 12:00-15:00

Last Name: ______________________________

First Name: ______________________________

E-mail: ______________________________

Signature: ______________________________

Instructions

1. Fill in the information above.
2. Please do not open the examination booklet until you are told to do so.
3. Do all your work in this test booklet. Show all your work.
4. Please no books, no notes, and no calculators.

1	2	3	4	5	6	7
/8	/12	/12	/12	/12	/5	/5
8	**9**	**10**	**11**	**12**	**13**	**Total**
/5	/10	/5	/5	/5	/4	/100

1. **[10 marks] Define the following terms:**

   (a) The limit of $f(x)$, as x approaches a

   (b) The derivative of a function f at a number a

   (c) Critical number of a function f

   (d) Antiderivative of a function f

2. **[12 marks] State the following theorems:**

   (a) The Intermediate Value Theorem

   (b) The Extreme Value Theorem

   (c) Fermat's Theorem

   (d) The Mean Value Theorem

3. [**12 marks**] **Give an example for the each of the following:**

   (a) Function $F = f \cdot g$ so that the limits of F and f at a exist and the limit of g at a does not exist.

   (b) Function that is continuous but not differentiable at a point.

   (c) Function with a critical number but no local maximum or minimum.

   (d) Function with a local minimum at which its second derivative equals 0.

4. [**12 marks**] **Evaluate the following limits:**

(a) $\lim_{x\to 0} x \sin\left(\frac{1}{x^2}\right)$

(b) $\lim_{x\to 0} \frac{x^3 \sin\left(\frac{1}{x^2}\right)}{\sin x}$

(c) $\lim_{x\to 0} x^{\tan x}$

5. [**12 marks**] **Find the derivative** $\mathbf{y}' = \frac{\mathbf{dy}}{\mathbf{dx}}$:

(a) $y = x^3 + 3^x + x^{3x}$

(b) $y = e^{-5x} \cosh 3x$

(c) $\tan^{-1}\frac{y}{x} = \frac{1}{2}\ln(x^2+y^2)$

(d) $y = \frac{x^5 e^{x^3}\sqrt[3]{x^2+1}}{(x+1)^4}$

6. **[5 marks]** A water tank is in the shape of a cone with vertical axis and vertex downward. The tank has radius 3 m and is 5 m high. At first the tank is full of water, but at time $t = 0$ (in seconds), a small hole at the vertex is opened and the water begins to drain. When the height of water in the tank has dropped to 3 m, the water is flowing out at 2 m^3/s. At what rate, in meters per second, is the water level dropping then?

   [**Note:** The volume of a cone with the radius r and the hight H is given by $V = \frac{\pi r^2 H}{3}$.]

7. **[5 marks]** Use the linear approximation to approximate $(63)^{2/3}$. Then use differentials to estimate the error.

8. **[5 marks]** Two horses start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed.

9. [**10 marks**] Sketch the graph of

$$y = 4x^{1/3} + x^{4/3}.$$

On your graph **clearly** indicate and label all intercepts, local extrema, and inflection points. **For full marks you have to show all your work.**

10. **[5 marks]** An open-topped cylindrical pot is to have volume 250 cm^2.The material for the bottom of the pot costs 4 cents per cm^2; that for its curved side costs 2 cents per cm^2. What dimensions will minimize the total cost of this pot?

    [**Note:** The area of a circle with the radius r equals $B = r^2\pi$; the circumference of the circle with the radius r equals $c = 2r\pi$; the volume of the cylinder is the product of the area of the base and the height.]

11. **[5 marks]**

(a) Show that Newton's method applied to the equation

$$\frac{1}{x} - a = 0$$

yields the iterative formula

$$x_{n+1} = 2x_n - a(x_n)^2$$

and thus provides a method for approximating the reciprocal $1/a$ without performing any divisions.

(b) Approximate $1/7$ by taking $x_0 = 0.12$ and calculating x_1.
[**Note:** Calculator gives $1/7 \approx 0.1428$.]

12. **[5 marks]** A particle starts from rest (that is with initial velocity zero) at the point $x = 10$ and moves along the x-axis with acceleration function $a(t) = 12t$. Find the resulting position function $x(t)$.

13. **[4 marks]** Sketch the curve

$$x = \sin^2 \pi t,\ \ y = \cos^2 \pi t,\ \ 0 \leq t \leq 2.$$

Clearly label the initial and terminal points and describe the motion of the point $(x(t), y(t))$ as t varies in the given interval.