Simon Fraser University
Department of Mathematics
Burnaby Campus and Surrey Campus

MATH 151-3, Fall 2004
Final Examination
December 15th, 2004, 8:30 – 11:30 am

Last Name (please print): _____

First Name (please print): _____

Student Number: _____

Section: _____

Instructor (please circle): E. Halmaghi N. Kouzniak P. Menz J. Yin

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 17 pages with a total of 11 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

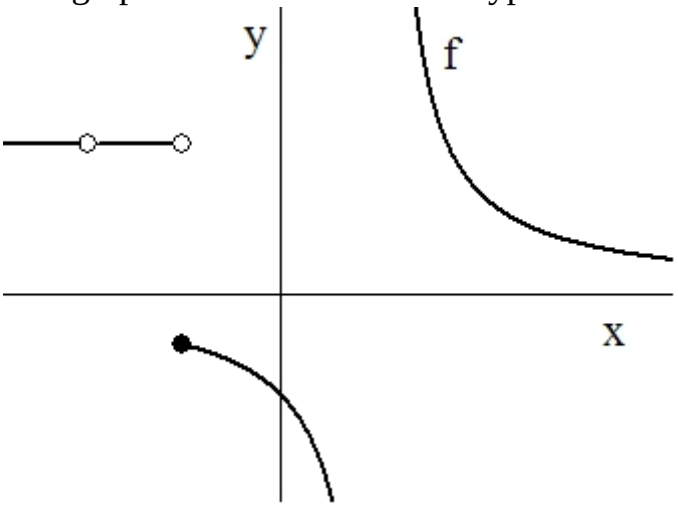
Question	Marks
1	**/10**
2	**/11**
3	**/20**
4	**/6**
5	**/6**
6	**/6**
7	**/8**
8	**/8**
9	**/5**
10	**/5**
11	**/15**
Total	**/100**

1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. Every correct answer will receive **1**, no penalties for incorrect answers. **[10 marks]**

a) ☐ The slope of the tangent line of $f(x)$ at the point $(a, f(a))$ is given by $\dfrac{f(a+h)-f(a)}{h}$.

b) ☐ Using the Intermediate Value Theorem it can be shown that $\lim\limits_{x\to 0} x\sin\dfrac{1}{x}=0$.

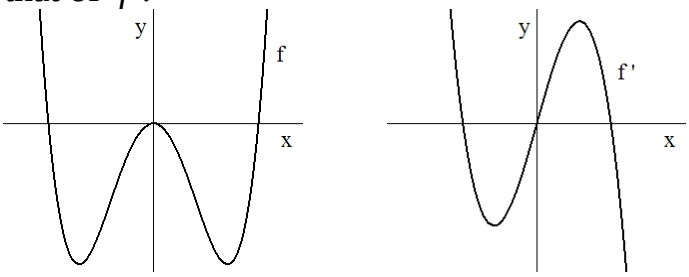
c) ☐ The graph below exhibits three types of discontinuities.



d) ☐ If $w=f(x), x=g(y), y=h(z)$ then $\dfrac{dw}{dz}=\dfrac{dw}{dx}\cdot\dfrac{dx}{dy}\cdot\dfrac{dy}{dz}$.

e) ☐ Suppose that on the open interval I, f is a differentiable function that has an inverse function f^{-1} and $f'(x)\neq 0$. Then f^{-1} is differentiable and $\left[f^{-1}(x)\right]'=\dfrac{1}{f'(f^{-1}(x))}$ for all x in the domain of f^{-1}.

f) ☐ Given the graph of f below to the left, the graph to the right must be that of f'.



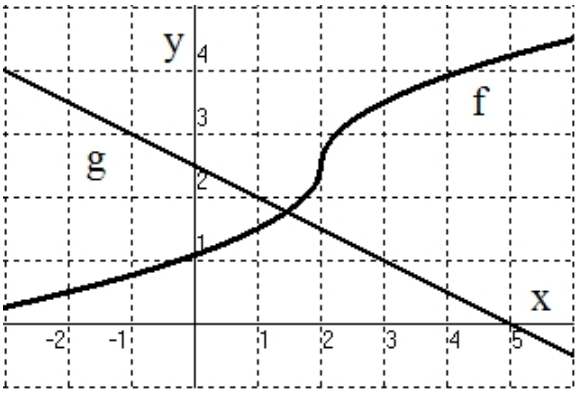
g) ☐ The conclusion of the Mean Value Theorem says that the graph of f has at least one tangent line in (a,b), whose slope is equal to the average slope on $[a,b]$.

h) ☐ The linear approximation $L(x)$ of a function $f(x)$ near the point $x=a$ is given by $L(x)=f'(a)+f(a)(x-a)$.

i) ☐ The graphs below are labeled correctly with possible eccentricities for the given conic sections.

e=1 e=0.8 e=0.5 e=0.1

j) ☐ Given $h(x)=g(f(x))$ and the graphs of f and g below then a good estimate for $h'(3)$ is $-\frac{1}{4}$.



2. Find the following limits: **[11 marks total]**

a) $\lim_{\theta \to \frac{\pi}{2}} \frac{\ln(\sin\theta)}{\cos\theta}$ **[4 marks]**

b) $\lim_{x \to \infty} \frac{1+3x}{\sqrt{2x^2+x}}$ **[4 marks]**

c) Let $F(x)=\dfrac{2x^2-3x}{|2x-3|}$. **[3 marks]**

i) Find $\lim\limits_{x\to 1.5^-} F(x)$.

ii) Find $\lim\limits_{x\to 1.5^+} F(x)$.

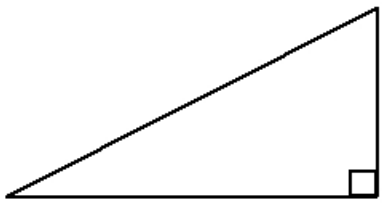
iii) Does $\lim\limits_{x\to 1.5} F(x)$ exist? Provide a reason.

3. Differentiation questions. **[4 marks each]**

a) Given $y = \tan(\cos^{-1}(e^{4x}))$, find $\frac{dy}{dx}$. Do not simplify your answer.

b) Given $y = \frac{1}{x} + \cos 2x$, find $\frac{d^5y}{dx^5}$. Simplify your answer.

c) Show that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$. You may use the right triangle below. If you do, label it.



d) Suppose f is a differentiable function such that $f(g(x)) = x$, and $f'(x) = 1 + [f(x)]^2$. Show that $g'(x) = \frac{1}{1+x^2}$.

e) Given $y = \dfrac{\sqrt{1+2x}\sqrt[4]{1+4x}\sqrt[6]{1+6x}\cdots\sqrt[100]{1+100x}}{\sqrt[3]{1+3x}\sqrt[5]{1+5x}\sqrt[7]{1+7x}\cdots\sqrt[101]{1+101x}}$, find y' at $x=0$.

4. Find the point(s) on the graph $y = x^3$ where the line through the point $(4,0)$ is tangent to y. **[6 marks]**

5. Use linear approximation to estimate $\sqrt{80}$. **[6 marks]**

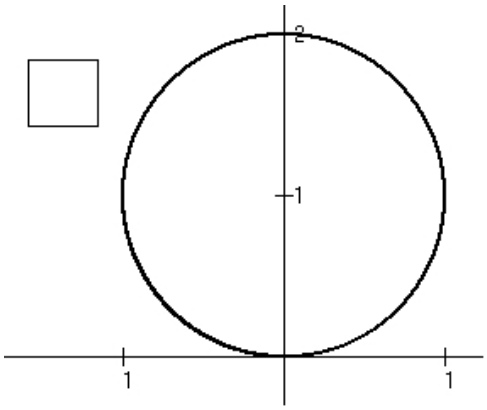
6. Derive the equation of the set of all points $P(x, y)$ that are equidistant from the point $A(1, 0)$ and the line $x = -5$. Provide a diagram with your work. Simplify the equation. **[6 marks]**

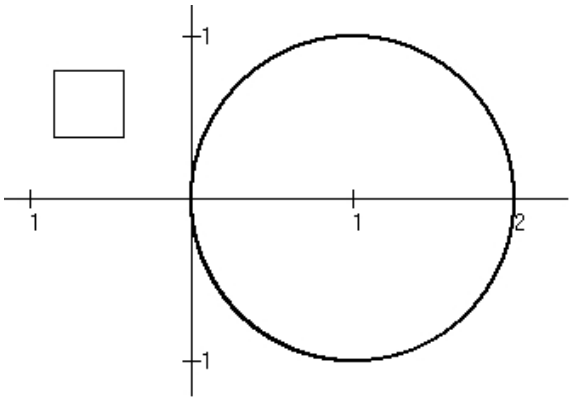
7. A farmer has 400 feet of fencing with which to build a rectangular pen. He will use all of an existing straight wall 100 feet long as part of one side of the perimeter of the pen. What is the maximum area that can be enclosed?
   [8 marks]

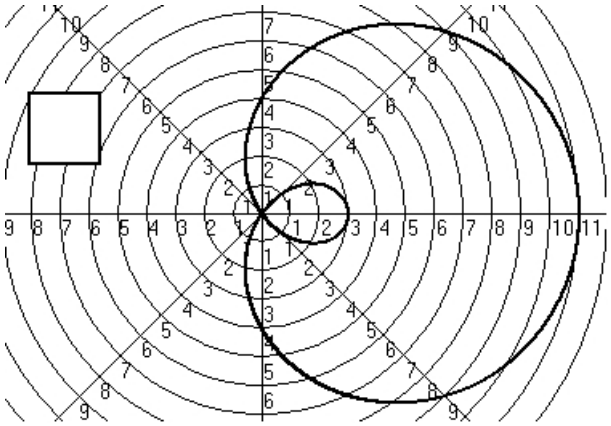
8. At noon of a certain day, ship A is 60 miles due north of ship B. If ship A sails east at 15 miles per hour and B sails north at 12.25 miles per hour, determine how rapidly the distance between them is changing 4 hours later. **[8 marks]**

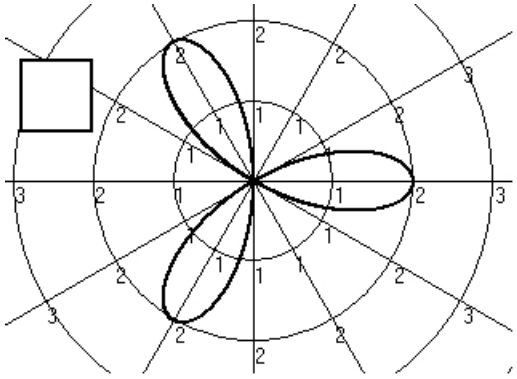
9. Label the five graphs below with the correct equation from the list below by placing the letter next to the equation in the square on each graph. **[5 marks]**

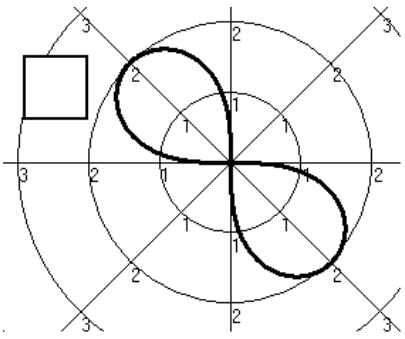
A. $r = 2\cos 3\theta$ B. $r^2 = -4\sin 2\theta$ C. $r = 2\sin\theta$
D. $r = 2\cos\theta$ E. $r = 4 + 7\cos\theta$











10. The parametric curve C is described by $x(t) = -e^{4t}, y(t) = e^{1-t}$, where $t \in \mathbb{R}$.
[5 marks total]

a) Find $\frac{dy}{dx}$ as a function of t directly from the above equations. Simplify your answer. **[2 marks]**

b) Find $\frac{d^2y}{dx^2}$ as a function of t. Simplify your answer. **[2 marks]**

c) Determine if C is concave up or concave down at $t = 0$. **[1 mark]**

11. Let $f(x)=\frac{4-4x}{x^2}$. Then $f'(x)=\frac{4(x-2)}{x^3}$ and $f''(x)=\frac{8(3-x)}{x^4}$. Determine the following. Show all your work. **[15 marks total]**

a) The domain of f. **[1 mark]**

b) The x- and y-coordinates of all intercepts. **[2 marks]**

c) All asymptotes. **[2 marks]**

d) The intervals on which f increases and the intervals on which f decreases. **[2 marks]**

e) The x- and y-coordinates of all critical points, each classified as a local maximum, minimum or neither. **[2 marks]**

f) The intervals on which f is concave up and the intervals on which f is concave down. **[2 marks]**

g) The x- and y-coordinates of all inflection points. **[1 mark]**

h) Sketch the graph of f using the information obtained in a) – g) and label all pertinent points and lines. **[3 marks]**