MATH 151-D200 Instructor: R. Pyke

Midterm 1, *Version 1*, September 30, 2008

Last Name:	
First Name:	
SFU Student email :	@sfu.ca

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.
5. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

Question	Score	Max
1		6
2		6
3		6
4		6
5		8
6		5
Total		37

(1) [Marks: 6] Use the Limit Laws to find the following limits. If the limit does not exist explain why, if the limit is infinite explain why. Show all steps of your work.

(a) $\lim_{x \to 0} \dfrac{x}{\sqrt{2+3x} - \sqrt{2}}$

(b) $\lim_{x \to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4} \right)$

(2) [Marks: 6] You are given the following information about the functions f, g and h (don't assume anything more than what is stated here!);

$$\lim_{x\to 2} f(x) = -4, \qquad \lim_{x\to -1} f(x) = 3, \qquad \lim_{x\to 0} f(x) = \frac{1}{2}$$

$$\lim_{x\to 2} g(x) = 7 \text{ and } g(2) = 3, \qquad \lim_{x\to -1} g(x) = 0$$

$$h(x) \text{ is continuous on } \mathbf{R} \text{ and } \quad h(2) = -1, \quad h(-1) = 5, \quad h(3) = -7$$

Evaluate the following expressions if possible (show your reasoning). If it is not possible explain why.

(a) $m(2)$ where $m(x) = f(x)h(x) + 1$

(b) $\lim_{x\to -1} \Big[\frac{f^2(x)}{x} - 2g(x)\Big]$

(c) $\lim_{x\to 2} \Big[\frac{f^2(x)}{x} - 2g(x)\Big]$

(d) $\lim_{x\to 2} \Big[f(x)h(x) + 1\Big]$

(e) $\lim_{x\to -1} \Big(h \circ f\Big)(x)$

(f) $\lim_{x\to -1} \dfrac{g(x)f(x)}{x+1}$

(3) [Marks: 6] The following function f is defined on all of $\mathbf{R}$ (all real numbers). Is it continuous on all of $\mathbf{R}$? If it is not continuous at some points determine the type of discontinuity at these points. Explain your answer completely.

$$f(x) = \begin{cases} 2\sin\left(\frac{\pi x}{2}\right) + \frac{1}{2} & x > -1 \\ 0 & x = -1 \\ \frac{-2x^2 - x + 1}{|1 - x^2|} & x < -1 \end{cases}$$

(4) [Marks: 6] **(a)** State the precise ε, δ definition of the limit of a function.

(b) Prove, using the ε, δ definition of limit, that

$$\lim_{x \to 1} (3x^2 + 2) = 5$$

(5) [Marks: 8] Consider the function

$$f(x) \;=\; \frac{x\sqrt[4]{3x^4+2x^2-x}}{2x^2-6}$$

(a) Find the limit $\lim\limits_{x\to-\sqrt{3}^{\,+}} f(x)$.

(b) Find the horizontal asymptote as $x \to -\infty$.

continued $\longrightarrow$

(c) Determine how the graph of $f(x)$ approaches each horizontal asymptote you found in part (b) (i.e., from above or below). (Remember, $\left|\frac{a}{x^n}\right| > \left|\frac{b}{x^m}\right|$ if $n < m$ and for any a, b as $x \longrightarrow \pm\infty$.)

(6) [Marks: 5] Use the definition of the derivative to find the equation of the tangent line to the graph of $f(x) = \sqrt{x^2 + 1}$ at the point on the graph with x-coordinate $= 1$.