MATH 151
Final Examination, December 17, 2005 R. Pyke

Last Name:	
First Name:	
SFU Student ID :	

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Once the test begins, please check that all pages are intact.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
4. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.
5. Duration of exam: 180 minutes. Marks for each question are indicated by [ ].

Some formulae:

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$$

$$\text{sphere}: \quad V = \frac{4}{3}\pi r^3, \quad S = 4\pi r^2$$

$$\text{circle}: \quad A = \pi r^2, \quad C = 2\pi r$$

$$\text{triangle}: \quad A = \frac{1}{2}bh$$

Question	**Mark**		**Question**	**Mark**
1	/8		7	/7
2	/9		8	/8
3	/5		9	/10
4	/4		10	/6
5	/4		11	/8
6	/7		**Total**	/76

(1) [8] Find the indicated limits. If the limit does not exist explain why.

(1a) $\lim_{x \to -2} \dfrac{x+2}{|x^2 - x - 6|}$

(1b) $\lim_{z \to 0} \dfrac{4 \tan 3z}{\sin 2z}$

(1c) $\lim_{x \to 0^+} (e^{2x} + x)^{\frac{1}{x}}$

(2) [9] Find the indicated derivatives of the following functions. You do *not* need to simplify your answers.

(2a) r' where $r = \left(\dfrac{\ln t}{t^2+1}\right)^2$

(2b) y' where $y = \left(\dfrac{1}{3}\right)^{2x} + (2x)^{1/3}$

(2c) y' where $(xy+1)^{5/2} = \tan(x+y^3)$

(3) [5] A ball is dropped from a height of 200 metres, 50 metres from a lamp that is also 200 metres tall. The ball's height above the ground at time t is $s(t) = 200 - 10t^2$ metres. Determine how quickly the ball's shadow (due to the lamp) is moving along the ground when the ball is half way to the ground.

(4) [4] Consider the following function;

$$f(x) = \begin{cases} cx + b & x < 0 \\ 3 + (x-2)^2 & x \geq 0 \end{cases}$$

Determine the values of c and b, if there are any, so that $f(x)$ is a differentiable function for all x.

(5) [4] Suppose $f(x)$ is a differentiable function on the interval $[1, 6]$. Suppose too that $f(5) = -1$ and $|f'(x)| \leq \frac{3}{2}$ for all x in $[1, 6]$. Use the Mean Value Theorem to determine <u>all</u> possible values of $f(1)$ and $f(6)$.

(6) [7] Determine whether or not the graphs of the following functions have any slant asymptotes. If they do, write down the equation of the asymptote (i.e., $y = mx + b$).

(a) $f(x) = \dfrac{2x^2 - 3}{3x + 1}$

(b) $f(x) = e^{2x} + \sqrt{2x^2 + x}$

(7) **(a)** [2] Use implicit differentiation to show that

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

(Hint: Consider the equation $\tan y = x$.)

(b) [2] Derive the equation of the linearization $L(x)$ of a function $f(x)$ at the point $x = a$. Illustrate with a sketch.

(c) [3] Use linear approximation to estimate $\tan^{-1}(1.2)$

(8) **(a)** [3] Find <u>all</u> the critical points of the function $y = |x| + \sin x$.

(b) [3] Find the absolute maximum and minimum values of the function $y = |x| + \sin x$ on the interval $[-\pi, 2\pi]$.

(c) [2] Does the function $y = |x| + \sin x$ have an absolute maximum or absolute minimum <u>on **R**</u>? (the entire real line). Explain.

(9) [10] Consider the function

$$f(x) = |x|^{1/4}(x^3 - 1)$$

Its derivatives are,

$$f'(x) = \begin{cases} \frac{1}{4}x^{-3/4}(13x^3 - 1) & x > 0 \\ -\frac{1}{4}(-x)^{-3/4}(13x^3 - 1) & x < 0 \end{cases}$$

and

$$f''(x) = \begin{cases} \frac{3}{16}x^{-7/4}(39x^3 + 1) & x > 0 \\ \frac{3}{16}(-x)^{-7/4}(39x^3 + 1) & x < 0 \end{cases}$$

Use these formulae to make a sketch of the graph of $f(x)$. Indicate critical points, local extrema, regions of increase/decrease, concavity, asymptotes, and points of inflection.

(10) [6] Find the length of the longest ladder that can be carried horizontally around the corner of the corridor shown here. (Hint: You are actually looking for the *minimum* length of something!)

(11) **(a)** [5] Sketch the parametric curve $(x(t), y(t))$ where the functions $x(t)$ and $y(t)$ are as below. Indicate on your sketch the points corresponding to the times $t = t_1,\ t = 0,\ t = t_2$.

(b) [3] Suppose that a curve given in polar coordinates by the equation $r = f(\theta)$ has even symmetry (so if you reflect the curve across the y-axis it looks the same). What can you say about the function $f(\theta)$? (Illustrate your answer by making a sketch of such an $f(\theta)$.)