SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Midterm 2

MATH 150 Fall 2006

Instructor: Dr. Mulholland

November 1, 2006, 8:30 – 9:20 a.m.

Name: Solutions (please print)

family name *given name*

SFU ID: ____________________

student number *SFU-email*

Signature: ____________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question . If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 8 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	15	
2	8	
3	4	
4	7	
5	6	
Total	40	

[3] **1.** (a) Compute $f'(x)$ if $f(x) = (x^5 - 2x^3 + 4)^7$. You do not need to simplify your answer.

$$f'(x) = 7(x^5 - 2x^3 + 4)^6 \cdot \frac{d}{dx}(x^5 - 2x^3 + 4) \quad \text{by Chain rule}$$

$$= 7(x^5 - 2x^3 + 4)^6 (5x^4 - 6x^2)$$

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[3] (b) Compute $g'(x)$ if $g(x) = \dfrac{\cos x}{1 + x^2}$. You do not need to simplify your answer.

$$g'(x) = \frac{-\sin x\,(1 + x^2) - \cos x \cdot (2x)}{(1 + x^2)^2} \quad \text{by Quotient rule}$$

$$= \frac{-(1 + x^2)\sin x - 2x\cos x}{(1 + x^2)^2}$$

[3] (c) Determine $h^{(37)}(t)$ if $h(t) = e^{-3t}$. (Compute the first few derivatives to find a pattern.)

$$h'(t) = -3e^{-3t}$$
$$h''(t) = (-3)^2 e^{-3t}$$
$$h'''(t) = (-3)^3 e^{-3t}$$
$$\vdots$$
$$h^{(n)}(t) = (-3)^n e^{-3t}$$

$$\text{so} \quad h^{(37)}(t) = (-3)^{37} e^{-3t}$$

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[3] (d) Find y' if $y = x^{2x}$. You do not need to simplify your answer.

$$\ln y = 2x \cdot \ln x$$
$$\frac{d}{dx} \qquad \frac{1}{y} y' = 2\ln x + 2x \cdot \frac{1}{x}$$
$$y' = y\left(2\ln x + 2\right)$$
$$y' = x^{2x}\left(2\ln x + 2\right)$$

[3] (e) Find $f'(x)$ if it is known that

$$\frac{d}{dx}[f(2x)] = x^2.$$

$$f'(2x) \cdot 2 = x^2 \qquad \text{by Chain rule.}$$

$$\Rightarrow \quad f'(2x) = \frac{x^2}{2}$$

setting $t = 2x$ i.e. $x = \frac{1}{2}t$

$$\Rightarrow \quad f'(t) = \frac{\left(\frac{1}{2}t\right)^2}{2} = \frac{t^2}{8}$$

Thus,

$$\boxed{f'(x) = \frac{x^2}{8}}$$

2. True or False. If True provide an explanation, if False give justification (for instance give an example for which the statement doesn't hold).

[2] (a) If $y = e^3$ then $y' = 3e^2$.

False, $y' = 0$ since e^3 is a constant.

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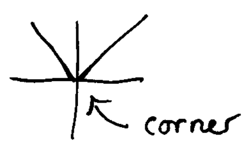
[2] (b) If f is continuous at $x = a$ then f is differentiable at $x = a$.

False. Consider $f(x) = |x|$.

Continuous @ $x=0$ but not differentiable @ $x=0$.



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[2] (c) If $f(x) = \dfrac{2x^3 + 5x - 1}{4x^3 - x^2 + 6x - 2}$, then $y = \frac{1}{2}$ is a horizontal asymptote.

True.
$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^3+5x-1}{4x^3-x^2+6x-2} = \frac{2}{4} = \frac{1}{2}.$$

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[2] (d) If $f(x)$ is an even function then $f'(x)$ is also an even function.

False. $f(x) = \cos(x)$ is even

but $f'(x) = -\sin(x)$ is odd

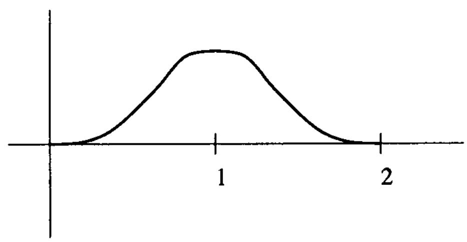
Another example:

$g(x) = x^2$ (even)

$g'(x) = 2x$ (odd)

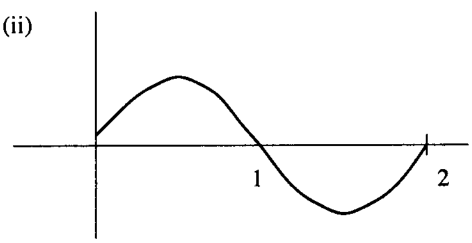
[3] **3.** (a) Below are the graphs of f, f' and f''. Determine which is the graph of f, which is f', and which is f'' and write your answer in the box next to the graph. You do NOT need to justify your answer.

(i)



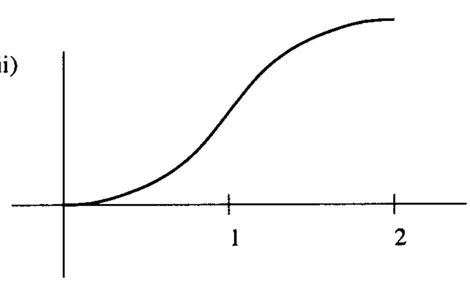
f'

(ii)



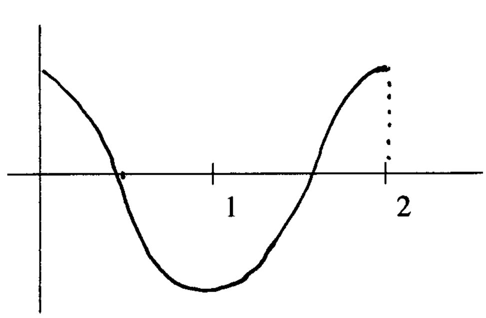
f''

(iii)



f

[1] (b) Sketch the graph of f'''.



4. A particle moves on a horizontal line so that its coordinate at time t is

$$x(t) = t^3 - 12t + 3, \quad t \geq 0.$$

[4] (a) Find the velocity and acceleration functions.

$v(t) = 3t^2 - 12$

$a(t) = 6t$

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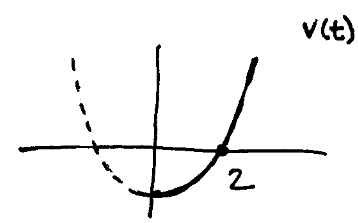
[2] (b) Determine when the particle is moving to the left and when it is moving to the right.



$$v(t) = 0 \Rightarrow 3t^2 - 12 = 0$$
$$\Rightarrow t^2 - 4 = 0$$
$$\Rightarrow t = 2, \cancel{-2}$$

when $t = 2$ velocity is 0 so particle is stopped.

when $0 \leq t < 2$ velocity is negative (see graph) so particle is moving to the left

When $t > 2$ velocity is positive so particle is moving to the right.

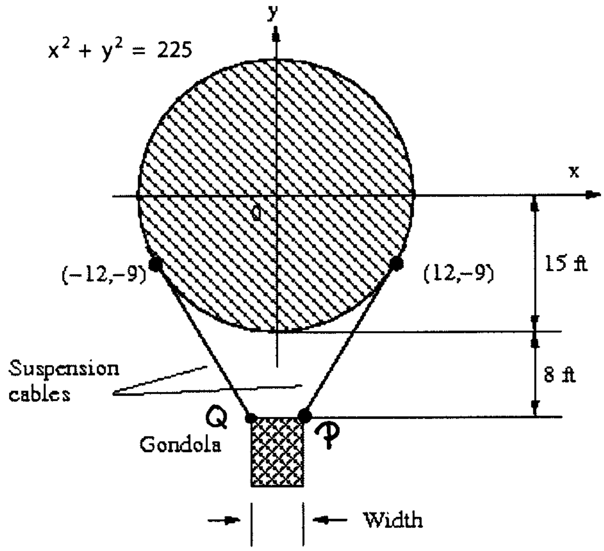
[1] (c) Determine when the particle is speeding up.

$a(t) = 6t$ which is always positive (since $t \geq 0$)

Speeding up when $v(t)$ & $a(t)$ have same sign

$\Rightarrow$ $\boxed{t > 2}$ or in interval notation (t, ∞).

[6] **5.** The designer of a 30-ft-diameter spherical hot-air balloon wishes to suspend the gondola 8 ft below the bottom of the balloon with suspension cables tangent to the surface of the balloon. Two of the cables are shown running from the top edges of the gondola to their points of tangency, $(-12, -9)$ and $(12, -9)$. How wide must the gondola be?



We want to find the tangent line at $(12,-9)$.

$$x^2+y^2=225$$

Implicit differentiation:

$$2x+2yy'=0$$

$$\Rightarrow \boxed{y'=\frac{-x}{y}}$$

At $(12,-9)$ the slope of the tangent line is

$$y'\Big|_{(12,-9)} = \frac{-12}{-9} = 4/3$$

Equation of tangent line at $(12,-9)$:

$$y+9 = 4/3\,(x-12)$$

$$\boxed{y=\frac{4}{3}x-25}$$

Point P has y-coordinate -23 and it's x-coord. is given by

$$-23 = 4/3\,x - 25 \quad \Rightarrow \quad x = 6/4 = 3/2.$$

By symmetry Q has coordinates $(-3/2,-23)$, so the width of the gondola is:

$$\text{width} = 2(3/2) = \boxed{3\text{ ft}}$$