

SIMON FRASER UNIVERSITY
DEPARTMENT OF MATHEMATICS

Final Exam

MATH 150 Fall 2007

Instructor: **(CIRCLE ONE)** Dr. Mulholland & Dr. Goddyn

December 13, 2007, 7:00 – 10:00 p.m.

Name: _____ (please print)
family name *given name*

SFU ID: _____
student number *SFU-email*

Signature: _____

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 11 questions on 13 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. Leave answers in "calculator ready" expressions: such as $3 + \ln 7$ or $e^{\sqrt{2}}$.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	12	
2	9	
3	9	
4	9	
5	9	
6	9	
7	9	
8	10	
9	10	
10	10	
11	0	
Total	96	

- [2] 1. (a) State the **Intermediate Value Theorem**, clearly identifying any hypotheses and the conclusion.

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- [2] (b) State the definition of **derivative** of a function f at a number a .

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- [2] (c) State the definition of a **critical number** of a function f .

- [2] (d) State the **Mean Value Theorem**, clearly identifying any hypotheses and the conclusion.

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- [2] (e) Give an example of a function f and a value $x = a$ such that $f''(a) = 0$ but f does *not* have an inflection point at $x = a$.

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- [2] (f) Give an example of a function defined everywhere on the closed interval $[0, 2]$ but with no absolute maximum on that interval.

Questions 2 - 7 are **Short-Answer Questions**. Each part of each question is worth 3 marks, but not all questions are of equal difficulty. Unless otherwise stated, it is not necessary to simplify your answers for these question. To receive full credit you must justify your answers.

[3] 2. (a) Evaluate $\lim_{x \rightarrow -\infty} \frac{(1 - x^2)(2 - x^2)}{5 + x - 3x^4}$.

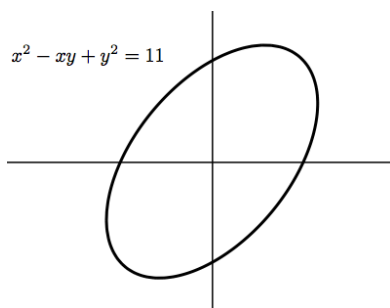
[3] (b) Evaluate $\lim_{x \rightarrow \pi/2^-} \left(x - \frac{\pi}{2}\right) \tan x$.

[3] (c) Evaluate $\lim_{x \rightarrow \infty} x^3 e^{-x+5}$.

- [3] 3. (a) Find the equation of the tangent line to the curve $y = (2 + x)e^{-x}$ at the point (0,2)

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- [3] (b) Find an equation of the slant asymptote to the curve $y = \frac{2x^3 + 3x^2 + 3x + 6}{x^2 + x + 1}$.

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- [3] (c) Find the point on the curve $x^2 - xy + y^2 = 11$ which lies in the first quadrant and has tangent line parallel to the line $y = -x - 2$.



[3] 4. (a) If $y = (\cos x)^x$, find the derivative dy/dx .

[3] (b) Show that the equation $x^4 = x + 1$ has **exactly** one solution in the interval $[1, 2]$.

[3] (c) Use Newton's Method with the initial approximation $x_1 = 1$ to find x_2 , the second approximation to the root of the equation $x^4 = x + 1$.

[3] 5. (a) Let $f(x) = x^3 \tan^2(5x)$. Find $f'(x)$.

[3] (b) Suppose f is a function satisfying $f(5) = 2$ and $f'(5) = 4$. Using a linear approximation to f at $x = 5$, find an approximation to $f(4.9)$.

[3] (c) Find a function f such that the curve $y = f(x)$ satisfies $\frac{d^2y}{dx^2} = 12x$, passes through the point $(0, 1)$, and has a horizontal tangent there.

- [3] 6. (a) Sketch the curve which is given by the parametric equations

$$x = 3 \sin(t), \quad y = 5 \cos(t), \quad 0 \leq t \leq 3\pi/2.$$

Clearly label the initial and terminal points and describe the motion of the point $(x(t), y(t))$ as t varies in the given interval (i.e. use arrows to indicate the direction the point is traveling).

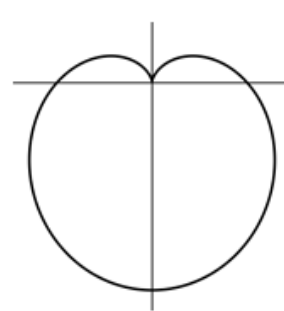
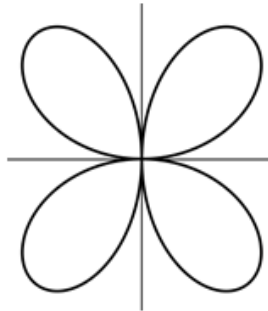
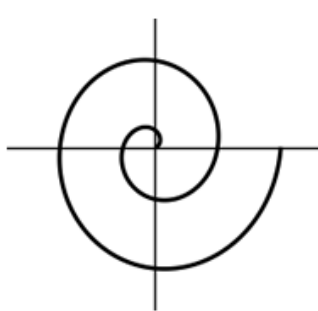
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- [3] (b) Find the slope of the tangent line to the curve in part (a) at the point $(x, y) = (\frac{3\sqrt{3}}{2}, \frac{5}{2})$.

[3] (c) The graphs of the equations

(i) $r = 3 \sin(2\theta)$

(ii) $r = 1 - \sin(\theta)$

are drawn below in polar coordinates. An extra graph is drawn as well. Match the equation with its corresponding graph and write your choice (either **(i)**, **(ii)** or **neither**) in the box under the graph.



7. A bank account initially contains \$1,000 and has an annual interest rate of 5%. Assume no additional deposits or withdrawals are made.

[3] (a) If the interest is compounded **annually**, write a formula for the balance at the end of t years.

[3] (b) If the interest is compounded **semi-annually**, write a formula for the balance at the end of t years.

[3] (c) If the interest is compounded **continuously**, write a formula for the balance at the end of t years.

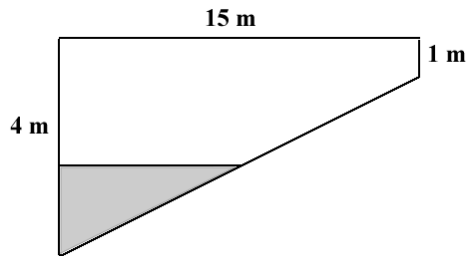
Long-Answer Problems: In questions 8-11, to receive full credit you must justify your answers and show all your work.

8. Consider the function $f(x) = x^3 e^{-x+5}$.

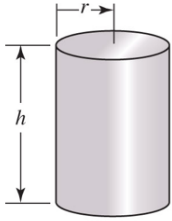
- [5] (a) Find and classify the critical points of f as either *local maximum*, *local minimum*, or *neither*.

- [5] (b) f has three inflection points, find them. Indicate the intervals where the graph of the function is concave-up and concave-down.

- [10] 9. A swimming pool is 15 m long and 5 m wide. Its depth varies uniformly from 1 m at the shallow end to 4 m at the deep end. Suppose that the pool is being filled at the rate of $10 \text{ m}^3/\text{min}$. At what rate is the depth of the water at the deep end increasing when the depth there is 2 m? (The figure shows a cross section of the pool.)



- [10] 10. A storage container is to be made in the form of a right circular cylinder and have a volume of $28\pi \text{ m}^3$. Material for the top of the container costs \$5 per square metre and material for the side and base costs \$2 per square metre. What dimensions will minimize the total cost of the container?



- [0] 11. **BONUS [4 points]** A number a is called a **fixed point** of a function f if $f(a) = a$. Show that if $f'(x) \neq 1$ for all real numbers x , then f has at most one fixed point.