SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Final Exam

MATH 150 Fall 2006

Instructor: Dr. Mulholland

December 14, 2006, 3:30 – 6:30 p.m.

Name: ______________________________ (please print)
family name *given name*

SFU ID: ______________________________
student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question . If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 12 questions on 13 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. Leave answers in "calculator ready" expressions: such as $3 + \ln 7$ or $e^{\sqrt{2}}$.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	12	
2	9	
3	9	
4	9	
5	9	
6	6	
7	10	
8	6	
9	10	
10	10	
11	6	
12	4	
Total	100	

[2] **1.** (a) State the definition of the **limit** of $f(x)$ as x approaches a.

---

[2] (b) State the definition of **derivative** of a function f at a number a.

---

[2] (c) State the definition of a **critical number** of a function f.

[2] (d) State the **Mean Value Theorem**, clearly identifying any hypotheses and the conclusion.

---

[2] (e) Give an example of a function that is continuous but not differentiable at a point.

---

[2] (f) Give an example of a function with a critical number but no maximum or minimum.

Questions 2 - 6 are **Short-Answer Questions**. Each question is worth 3 marks, but not all questions are of equal difficulty. Unless otherwise stated, it is not necessary to simplify your answers for these question.

[3] **2.** (a) Evaluate $\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2}$

---

[3] (b) Evaluate $\lim_{x\to \infty} \frac{x^2-1}{x^2-3x+2}$

---

[3] (c) Calculate the derivative of $f(x) = \sin^{-1}(x^2)$. [*Note*: Another notation for $\sin^{-1}$ is $\arcsin$.]

[3] **3.** (a) The curve $y = xe^{-2x}$ has one inflection point. Find the x coordinate of this point.

---

[3] (b) Find an equation of the slant asymptote to the curve $y = \dfrac{x^3 + x^2}{x^2 + 1}$.

---

[3] (c) Find an equation of the tangent line at $(0, 1)$ to the curve $x^3 + y^3 = \sin x + 1$.

[3] **4.** (a) Find a number x_0 between 0 and $\pi/2$ such that the tangent line to the curve $y = \cos x$ at $x = x_0$ is parallel to the line $y = -x/2$.

---

[3] (b) If $y = x^{\sin x}$, find the derivative dy/dx.

---

[3] (c) Using linear approximation, estimate $f(4.1)$, given that $f(4) = 3$ and $f'(x) = \sqrt{x^2 + 9}$.

[3] **5.** (a) Evaluate $\lim_{x \to 0} \dfrac{e^x - 1}{\sin(2x)}$.

---

[3] (b) If $f'(x) = 6x^2 + \sin x$ and $f(0) = 1$ find f.

---

[3] (c) The function f is defined by

$$f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ Ax + B & \text{if } x \geq 1 \end{cases}$$

where A and B are constants. For what values of A and B is $f(x)$ differentiable for all x?

[3] **6.** (a) Sketch the curve which is given by the parametric equations

$$x = \cos(\pi t), \quad y = \sin(\pi t), \quad 1 \leq t \leq 2.$$

Clearly label the initial and terminal points and describe the motion of the point $(x(t), y(t))$ as t varies in the given interval (i.e. use arrows to indicate the direction the point is traveling).

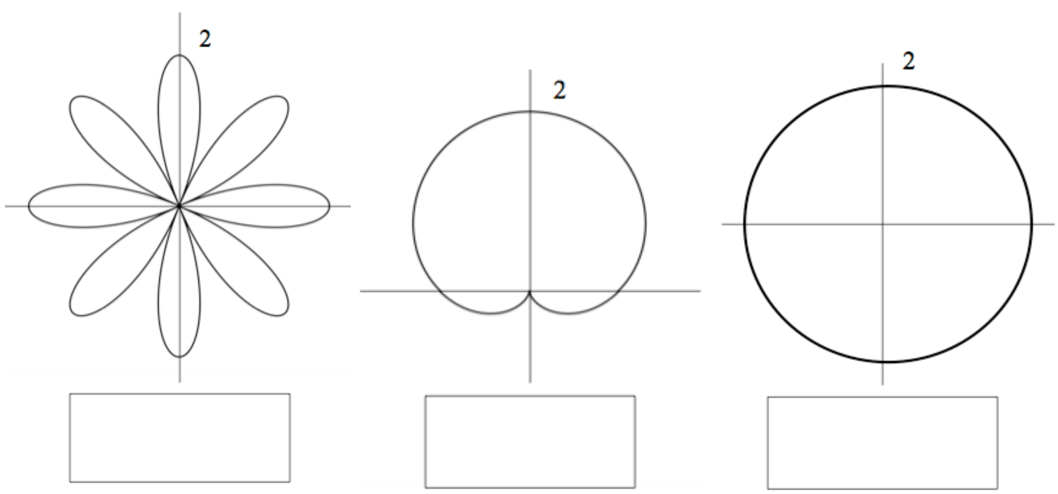
---

[3] (b) The graphs of the equations

(i) $r = 1 + \sin\theta$

(ii) $r = 2\cos(4\theta)$

are drawn below in polar coordinates. An extra graph is drawn as well. Match the equation with its corresponding graph and write your choice (either (**i**), (**ii**) or **neither**) in the box under the graph.



Full-Solution Problems: In questions 7-12, justify your answers and **show all your work**

[10] **7.** A turkey is put into an oven that has a constant temperature of $200°$C. A thermometer embedded in the turkey registers its temperature. When the turkey is put into the oven, the thermometer reads $20°$C, and 30 minutes later it reads $30°$C. The turkey will be ready to each when the thermometer reads $80°$C. How many minutes after being put into the oven will the turkey be ready to eat? Assume that the turkey's temperature satisfies Newton's law of cooling/heating.

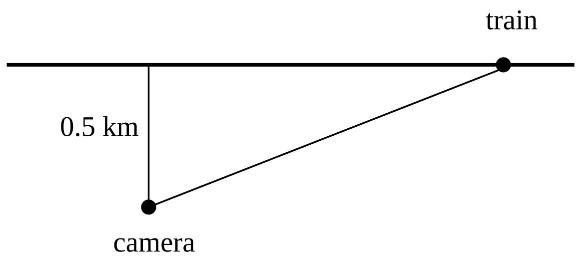
[4] **8.** (a) Show that Newton's Method applied to the equation $x^2 - a = 0$ yields the iterative formula

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

and thus provides a method for approximating the square root $\sqrt{a}$ which uses only addition and multiplication and division.

[2] (b) Approximate $\sqrt{3}$ by taking $x_1 = 3/2$ and calculating x_2.

[10] **9.** A high speed train is traveling at 3km/min along a straight track. The train is moving away from a movie camera which is located 0.5km from the track. The camera keeps turning so as to always point at the front of the train. How fast (in radians per minute) is the camera rotating when the front of the train is 1 km from the camera?



[10] **10.** An open-topped box with a square base is to have volume 300 cm^3. The material for the bottom of the box costs 4 cents per cm^2 and the material for the sides of the box costs 2 cents per cm^2. What dimensions will minimize the total cost of the box?

[6] **11.** Find the derivative of

$$\frac{1}{1-x^2}$$

using the definition of the derivative. No marks will be given for the use of differentiation rules.

[4] **12.** Suppose $f(x)$ is a function that is differentiable for all x. Let $g(x)$ be the new function defined by $g(x) = f(x) + f(1 - x^2)$. Prove that $g'(c) = 0$ for some real number c in the interval $(0, 1)$.