Department of Mathematics

MATH 100, PRECALCULUS
MIDTERM 2

Wednesday March 15, 2006
11.30 – 12.20

Last Name _____

Given Name(s) _____

Student Number _____

Signature _____

INSTRUCTIONS

1. Do **NOT** open this booklet until permission is given.
2. Calculators are **NOT** permitted.
3. Please formulate and motivate your answers.
   Well-phrased and complete explanations are more important than just an answer. It should be clear how you obtained your answer.
4. If the space provided for the answer is insufficient, please use the back side of the **PREVIOUS** page. Clearly mark which question you are answering in that case.
5. If a question is unclear or appears to contain an error, please ask for clarification.
6. The maximum mark for this exam is 40. In front of each question, the maximal mark for that question is given.
7. This exam consists of **9 PAGES** (including this one), and contains one double-sided **GREEN** formula sheet.
8. Any notes you make on the formula sheet are **NOT** considered to be part of your exam.

Answers

1. QUESTION

In this question we let f be the quadratic function

$$f(x) = x^2 + 6x - 10.$$

[4] (a) Compute the vertex of the graph of f.

ANSWER

$X = \frac{-b}{2a} = \frac{-6}{2} = -3$

$f(-3) = (-3)^2 + 6 \cdot (-3) - 10 = -19$

vertex $= (-3, -19)$

[2] (b) What is the axis of symmetry of this parabola?

ANSWER

$X = -3$, vertical axis through the vertex

2. QUESTION

[4] (a) Divide $2x^2 - 7x + 4$ by $x + 3$ using long division. State clearly what the quotient is and what the remainder is.

ANSWER

$$x+3 \;/\; 2x^2 - 7x + 4 \;\backslash\; 2x - 13 \quad \text{quotient}$$

$$(2x^2 + 6x) -$$

$$-13x + 4$$

$$(-13x - 39) -$$

$$43 \quad \text{remainder}$$

[2] (b) Give the equation of the slant asymptote of the graph of $\frac{2x^2-7x+4}{x+3}$? Explain briefly.

ANSWER

$y = 2x - 13$

It is the quotient of $2x^2 - 7x + 4$ divided by $x+3$ (see (a)).

3. QUESTION

In this question we take

$$f(x) = \frac{x^2 - 4}{x^2 + 2x}.$$

[3] (a) Factor the numerator and the denominator of f and simplify f.

ANSWER

$X^2-4 = (x-2)\cdot(x+2)$

$x^2+2x = x(x+2)$

$f(x) = \frac{x^2-4}{x^2+2x} = \frac{(x-2)(x+2)}{x(x+2)} = \frac{x-2}{x}$ $x \neq 0, -2$

[2] (b) Give the domain of f in set notation. Indicate why this is the domain.

ANSWER

The domain of f is everywhere where its denominator is <u>not</u> 0 (of the original function)

Domain = $\{x \in \mathbb{R} \mid x \neq 0, x \neq -2\}$

[1] (c) Find the zero(es) of f.

ANSWER Find those x such that $f(x)=0$. Thus

$x^2-4=0 \Rightarrow x=2, x=-2$ however

$x=-2$ is not in the domain

$x=2$ is the only zero

[2] (d) Compute $f(-3)$, $f(-1)$, $f(1)$ and $f(3)$.

ANSWER

$$f(-3) = \frac{-3-2}{-3} = \frac{5}{3}$$

$$f(-1) = \frac{-1-2}{-1} = 3$$

$$f(1) = \frac{1-2}{1} = -1$$

$$f(3) = \frac{3-2}{3} = \frac{1}{3}$$

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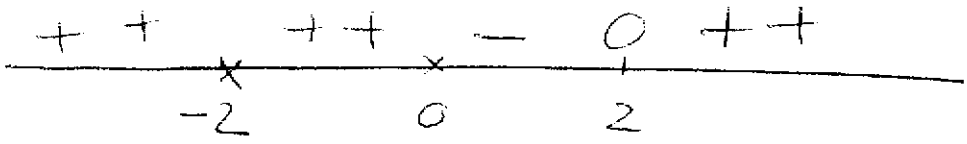
[2] (e) Write the solution set of the inequality $\frac{x^2-4}{x^2+2x} \geq 0$ in interval notation.(Hint: you might want to use a number line.)

ANSWER



$(-\infty, -2)$ or $(-2, 0)$ or $[2, \infty)$

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4. QUESTION

In this question all angles are in **radians**.

[4] (a) Compute and simplify $\tan(-\frac{\pi}{3})$. (Do not leave any square roots in the denominator, if there are any.)

ANSWER

$$\tan\left(-\frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

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[2] (b) Compute $-3\sin^2(\frac{3\pi}{4}) - 3\cos^2(\frac{3\pi}{4})$.

ANSWER

$$-3\sin^2\left(\frac{3\pi}{4}\right) - 3\cos^2\left(\frac{3\pi}{4}\right)$$

$$= -3\left(\sin^2\left(\frac{3\pi}{4}\right) + \cos^2\left(\frac{3\pi}{4}\right)\right)$$

$$= -3 \cdot 1$$

$$= -3.$$

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5. QUESTION

In this question all angles are in **radians**.

[2] (a) Compute the period of $y = \cos(2x)$.

ANSWER

Period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

$y = A\cos(Bx - C)$

[4] (b) What is the phase shift of $y = \cos(2x + \pi)$?

ANSWER

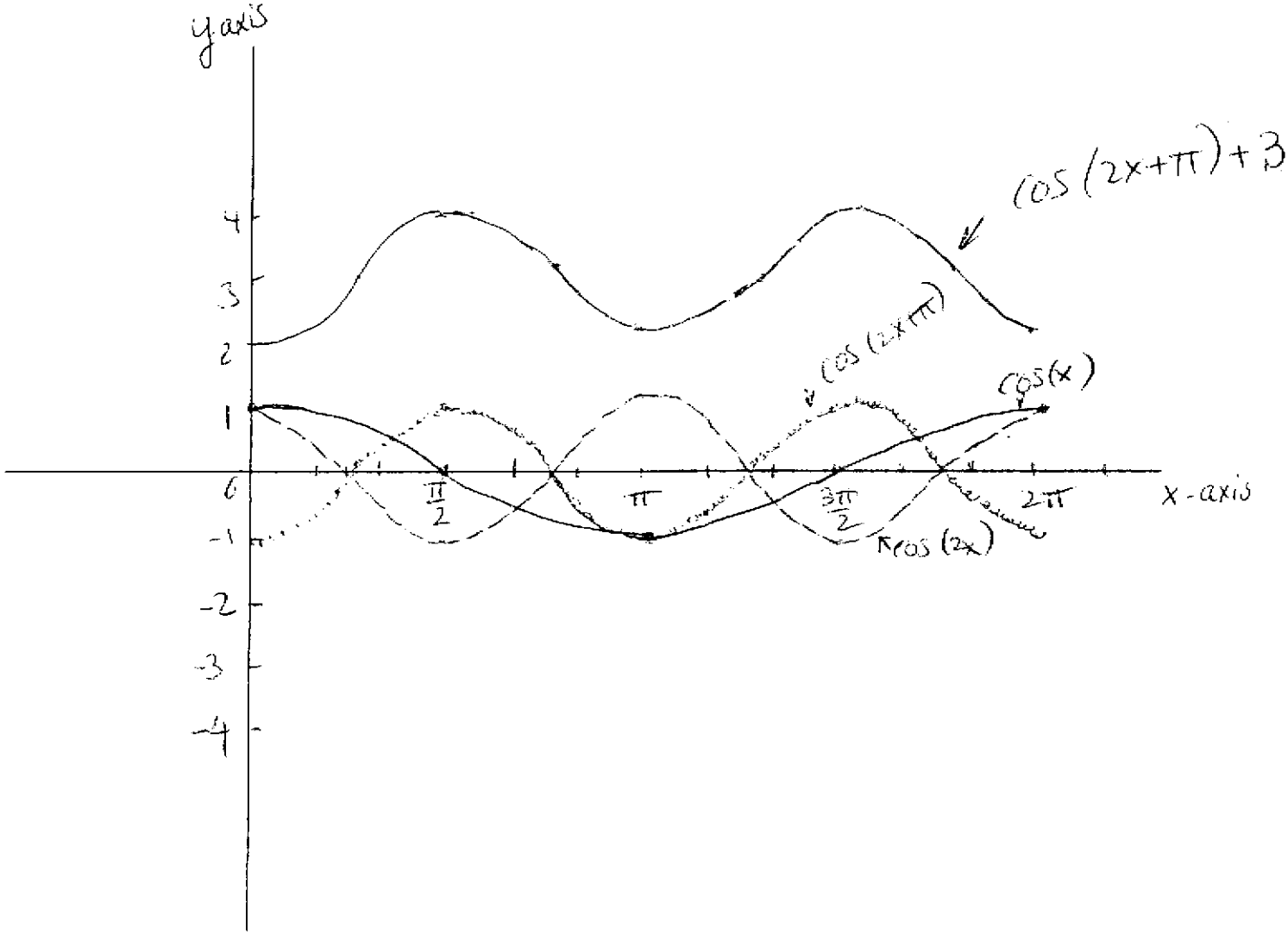
$y = \cos(2x + \pi)$

$= \cos(2x - (-\pi))$

Phase shift is $\frac{C}{B} = \frac{-\pi}{2}$.

[4] (c) Graph $y = \cos(2x - \pi) - 3$ for $0 \leq x \leq 2\pi$, by successively graphing $y = \cos(x)$, $y = \cos(2x)$, $y = \cos(2x + \pi)$ and $y = \cos(2x - \pi) + 3$ on the axes below.
Do not forget to put scales on the axes.
Make sure that the minima, maxima, and zeroes of $\cos(x)$, are clearly visible.
Label your graphs clearly.

ANSWER



DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	6	
2	6	
3	10	
4	6	
5	10	
Total	38	

Student number