Simon Fraser University
Math 100

Midterm 2- Exam 1

Date: November 7, 2007
Time: 11:30 - 12:20

Last Name (print): Key **First Name** ____________

Signature: ____________ **SFU Email ID:** ____________

Instructions:

1. Do not open this exam until instructed to do so.
2. **No calculators, notes or books are allowed.**
3. When presenting a final answer for your solution, calculator-ready expressions will be given full credit.
4. Show all your work. **No credit** will be given for an answer without the correct explanation and accompanying work.
5. Answer the questions in the space provided. Continue on the back of the previous page if necessary.

Question	Mark	Maximum
1		9
2		12
3		8
4		7
5		10
6		4
Total		50

1. Given the function $f(x) = -3x^4 + 12x^2 - 8$, answer the following questions:

[2 pts] (a) Describe the end behavior of $f(x)$

f(x) falls to the right & left

$\swarrow \qquad \searrow$

[1 pts] (b) Find the y-intercept of $f(x)$

$f(0) = -8$

$(0, -8)$

[2 pts] (c) Determine if f is even, odd or neither.

f is a polynomial & all exponents are even, so f is even

[4 pts] (d) Find the x-intercepts of the graph of $f(x)$.

$f(x) = -3(x^2)^2 + 12(x^2) - 8$

$x^2_{1,2} = \dfrac{-12 \pm \sqrt{12^2 - 4(3)(8)}}{-6} = \dfrac{-12 \pm \sqrt{12(12-8)}}{-6} = \dfrac{12 \pm 4\sqrt{3}}{6}$

$x^2_{1,2} = 2 \pm 2\dfrac{\sqrt{3}}{3}$

Zeroes: $\sqrt{2 + \dfrac{2\sqrt{3}}{3}}$; $-\sqrt{2 + \dfrac{2\sqrt{3}}{3}}$

$\sqrt{2 - \dfrac{2\sqrt{3}}{3}}$; $-\sqrt{2 - \dfrac{2\sqrt{3}}{3}}$

2. Given the rational function $h(x) = \dfrac{(3x-6)(x^2-1)}{x^2-2x-3}$ answer the following questions about h.

[7 pts] (a) Determine if the graph of h has any holes or asymptotes. If so, give the equation of the asymptotes and state the location of the holes.

$$h(x) = \frac{3(x-2)(x-1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

$x \neq -1$ $h(x)$ has a hole at $x=-1$.

h has a vertical asymptote at $x=3$.

$3(x-2)(x-1) = 3x^2 - 9x + 6$

$h(x) = \dfrac{P(x)}{Q(x)}$ ← deg. 3 ← deg. 2.

$$\begin{array}{r|l} & 3x \\ \hline x-3 & 3x^2 - 9x + 6 \\ & 3x^2 - 9x \\ \hline & \phantom{3x^2 - 9x +{}} 6 \end{array}$$

h has a slant asymptote with equation $y = 3x$.

[3 pts] (b) Give the x and y-intercepts, if they exist.

$0 \in$ Domain (h). so $h(0) = \frac{6}{-3} = -2$.

y-intercept is $(0,-2)$.

x-intercepts at $(2,0)$, $(1,0)$.

[2 pts] (c) State the domain of $h(x)$.

$\mathbb{R} \setminus \{3,-1\}$ or $x \neq -1, 3$ or

$\{x \in \mathbb{R} ; x \neq -1, 3\}$ or $(-\infty,-1) \cup (-1,3) \cup (3,\infty)$

3. Decide if the following statements are true or false. In each case mark (circle or cross) the correct answer and give a reason for your decision. **NO credit** will be given for an answer without explanation.

[1 pts] (a) The rational function $g(x) = \dfrac{x^2 - 5x + 1}{x(x-2)(x+3)}$ admits a decomposition into partial fractions of the form:

$$\frac{x^2 - 5x + 1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

[TRUE] **[FALSE]**

The denominator is a product of linear factors with no repetition.

[1 pts] (b) $x = 3$ is a zero of multiplicity 2 of $p(x) = -3x^2(x+3)^4(x-3)^2$

[TRUE] **[FALSE]**

$(x-3)^2$ is a factor of $p(x)$
$(x-3)^3$ is Not a factor of $p(x)$

[1 pts] (c) The angles $\alpha = 30°$ and $\beta = \dfrac{5\pi}{6}$ are complementary.

[TRUE] **[FALSE]**

$\alpha = \dfrac{\pi}{6} \Rightarrow \alpha + \beta = \pi$

α, β are suplementary.

[2 pts] (d) The graph of a rational function cannot have more than one horizontal asymptote.

[TRUE] [FALSE]

Such an asymptote has equation $y = \frac{a_n}{b_n}$ with $f(x) = \frac{a_n x^n + \cdots + a_0}{b_n x^n + \cdot \; + b_0}$. There can be only one value for $\frac{a_n}{b_n}$

[3 pts] (e) There is a value $0 < c < 1$ for which the functions $f(x) = x^3 - 2x^2 + 2$ and $g(x) = 3\sin\left(\frac{\pi}{2}x\right)$ are equal.

[TRUE] [FALSE]

f, g & $f-g$ are continuos, so we can use the int. value Thm.

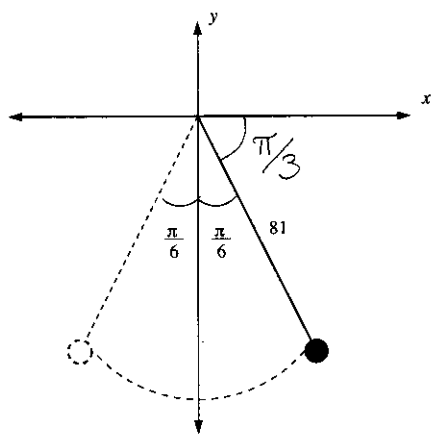
$f(0) = 2$ $\quad g(0) = 3 \quad (g-f)(0) = 1$

$f(1) = 1 \quad g(1) = 0 \quad (g-f)(1) = -1$

$\Rightarrow \exists \; c \in (0,1)$ s.t $(g-f)(c) = 0$

$\Leftrightarrow g(c) = f(c)$ as required.

4.



A ball at the end of a string hangs from the ceiling and forms a pendulum that swings right and left. The length of the pendulum is 81 cm and its motion describes an angle of $\pi/3$, as indicated in the picture.

[3 pts] (a) What is the length of the arc subtended by one full swing (all the way from right to left) of the pendulum?

The angle is $\vartheta = \frac{\pi}{3}$ The arc length is, then:

$s = r\vartheta = 81 \cdot \frac{\pi}{3} = 27\pi$.

[2 pts] (b) Find the coordinates of the position of the ball when the pendulum is furthest to the right (the black ball on the picture).

coordinates are: $(81\cos(-\frac{\pi}{3}), 81\sin(-\frac{\pi}{3}))$

$= \left(\frac{81}{2}, -\frac{81\sqrt{3}}{2}\right)$

[2 pts] (c) If it takes half a second for the pendulum to make one full swing from right to left, what is the linear speed of the ball?

l. speed $= \frac{27\pi}{\frac{1}{2}} = 54\pi$

The speed is 54π cm/s.

5. Consider the function $f(t) = -3\cos\left(2t + \frac{3\pi}{2}\right)$.

[1 pts] (a) Find the amplitude of f.

$|A| = 3$

[1 pts] (b) Determine the phase-shift of f.

$\frac{\frac{3\pi}{2}}{2} = \frac{3\pi}{4}$

[1 pts] (c) Calculate the period of f

$\frac{2\pi}{2} = \pi$.

[3 pts] (d) Find the zeros of f

$\cos t = 0$ for $t = \frac{\pi}{2} \pm n\frac{\pi}{4}$ $\frac{1}{2}$ (period)

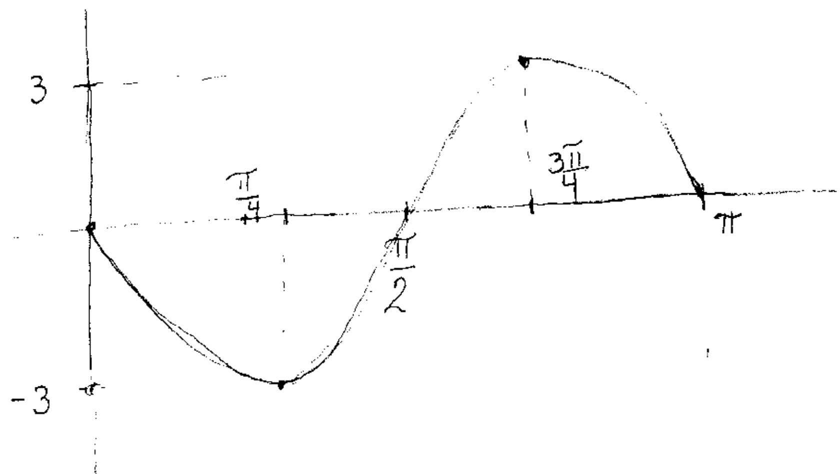
$2t + \frac{3\pi}{2} = \frac{\pi}{2}$

$2t = -\pi$

$t = -\frac{\pi}{2}$ $\pm n(\frac{1}{2}$ period of $f(t))$

$t = -\frac{\pi}{2} \pm n\left(\frac{\pi}{2}\right)$

[4 pts] (e) Sketch one cycle of the graph of f. Label all important points.



[4 pts] 6. List all the possible candidates for the rational zeros of the polynomial $p(x) = -3x^5 + x^4 - 2x^2 + 10$. You **do not** have to decide whether any of them is actually a zero.

$10 = 1 \cdot 2 \cdot 5$

$3 = 1 \cdot 3$

$\frac{p}{s} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3} \pm 10, \pm \frac{10}{3}$