Simon Fraser University
Math 100

Midterm 2

Solutions

Date: November 8, 2006

Time: 11:30 - 12:20pm

Last Name (print): ____________________ **First Name** ____________________

Signature: ____________________ **SFU Email ID:** ____________________

Instructions:

1. Do not open this exam until instructed to do so.
2. No calculators, notes or books are allowed.
3. When presenting a final answer for your solution, calculator-ready expressions will be given full credit.
4. Show all your work. **No credit** will be given for an answer without the correct accompanying work.
5. Answer the questions in the space provided. Continue on the back of the previous page if necessary.

Question	Mark	Maximum
1		8
2		8
3		9
4		11
5		5
6		5
Total		46

1. The annual yield per walnut tree is fairly constant at 60 pounds per tree when the number of trees per acre is 20 or fewer. For each additional tree over 20, the annual yield per tree for all trees on the acre decreases by 2 pounds due to overcrowding.

[3 pts] (a) If x is the number of trees planted in the acre, find an equation for the function $f(x)$ that gives the total yield for the acre when $x \geq 20$.

$$\begin{aligned} f(x) &= x\left[60 - 2(x-20)\right] \\ &= 60x - 2x^2 + 40x \\ &= -2x^2 + 100x \end{aligned}$$

[4 pts] (b) How many walnut trees should be planted per acre to maximize the annual yield for the acre?

$$\begin{aligned} -2x^2 + 100x &= -2(x^2 + 50x) \\ &= -2(x^2 - 50x + 25^2 - 25^2) \\ &= -2(x-25)^2 + 2(25)^2 \\ &= -2(x-25)^2 + 1250 \end{aligned}$$

Max yield when $x = 25$.

(Vertex at $(25, 1250)$).

[1 pts] (c) What is the maximum number of pounds of walnuts per acre?

Max # of pounds is $f(25) = 1250$.

2. Let $f(x) = -x^5 + 6x^4 + 16x^3$.

[1 pts] (a) Determine the end behaviour of the graph of $f(x)$.

Rises on the left
Falls on the right.

[5 pts] (b) Find the x-intercepts and determine if the graph crosses the x-axis or touches and turns around.

Solve: $-x^5 + 6x^4 + 16x^3 = 0$

$-x^3(x^2 - 6x - 16) = 0$

$-x^3(x-8)(x+2) = 0$

Intercepts at: $x = 0, 8, -2$.
All roots have odd multiplicity, so the graph crosses at $x = 0$, $x = 8$, $x = -2$.

[2 pts] (c) Determine if the graph has any symmetries. Explain.

$f(-x) = -(-x)^5 + 6(-x)^4 + 16(-x)^3$

$= x^5 + 6x^4 - 16x$

$\left.\begin{array}{l} f(-x) \neq f(x) \\ f(-x) \neq -f(x) \end{array}\right\}$ No symmetries.
($f(x)$ is neither even nor odd)

4. Let $f(x) = 2x^3 - 4x^2 + 6$

[3 pts] (a) Find $f(7)$ **using the remainder theorem.**

$$\begin{array}{r|rrrr} 7 & 2 & -4 & 0 & 6 \\ & & 14 & 70 & 490 \\ \hline & 2 & 10 & 0 & 496 \end{array}$$

$f(7) = 496.$

[5 pts] (b) Solve the inequality $f(x) \leq 2x^3 - 2x^2 + 2$.

$\cancel{2x^3} - 4x^2 + 6 \leq \cancel{2x^3} - 2x^2 + 2$

$-2x^2 + 4 \leq 0$

$-2(x^2 - 2) \leq 0 \rightarrow$ Solve: $-2(x - \sqrt{2})(x + \sqrt{2}) = 0$

$-2 \quad -\sqrt{2} \quad 0 \quad \sqrt{2}$

Sol. set: $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty$

Test: $-2 \qquad 0 \qquad 2$

Value: $-2(-2-\sqrt{2})(-2+\sqrt{2}) < 0 \quad -2(-\sqrt{2})(\sqrt{2}) > 0 \quad -2(2-\sqrt{2})(2+\sqrt{2}) < 0$

[3 pts] (c) Is there a solution to $f(x) = 0$ in the interval $(-2, 0)$? Yes, no, explain.

$f(-2) = 2(-2)^3 - 4(-2)^2 + 6 = -16 - 16 + 6 < 0$

$f(0) = 0 - 0 + 6 > 0$

By intermediate value thm. Yes

There is a sol.

[5 pts] 5. Solve the equation $\ln(2x-2)+\ln x = \ln(2x+30)$.

$\ln(2x-2)x = \ln(2x+30)$

$(2x-2)x = 2x+30$

$2x^2-2x-2x-30=0$

$2(x^2-2x-15)=0$

$2(x-5)(x+3)=0$

Possible sol. $x=5$, $x=-3$

a) $x=-3$ is not a sol. since $\ln(-3)$ is not defined.

b) $x=5$: $2(5)-2>0$; $5>0$; $2(5)+30>0$

So $x=5$ is the only solution.

[5 pts] 6. Find a model for the decay of a radioactive substance that has a half-life of 1.3 billion years. Give a "calculator ready" expression for the value of any constants involved.

General model for exp. decay:

$f(t) = A_0 e^{-Kt}$ t : time (in billion years, for this case).

$f(1.3) = \frac{A_0}{2} = A_0 e^{-K\,1.3}$

$\frac{1}{2} = e^{-1.3K}$

$\ln(\frac{1}{2}) = -1.3K$

$K = -\frac{\ln(1/2)}{1.3}$